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SIMULATION OF TURBULENT CONVECTIVE HEAT TRANSFER TO WATER IN A VERTICAL PIPE

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Abstract: The present work contains the results of direct numerical simulations using five different k~ϵ turbulence models of the experiments performed on buoyancy-influenced descending flow of water in a heated vertical tube. The simulations have been performed using a variable property formulation of the governing equations. Of the models considered, it has been found that only one, namely the Launder and Sharma model is capable of reproducing the observed influence of buoyancy satisfactorily.

NOMENCLATURE

c_p	specific heat capacity at constant pressure
C_1, C_2	constants in modelled dissipation equation
C_μ	constant in constitutive equation of eddy viscosity model
D,E	terms in low-Reynolds-number k-equation
f_1, f_2, f_3	functions in dissipation equation
f_μ	function in constitutive equation of k~ϵ model
g	acceleration due to gravity
h	enthalpy
k	turbulence kinetic energy
Nu	Nusselt number, $q_wD/(T_w-T_b)\lambda$
p	pressure
Pr	Prandtl number, $c_p\mu/\lambda$
r,z	cylindrical polar coordinates
Re	Reynolds number, $\rho W_bD/\mu$
V,W	mean velocity components in r,z directions

GREEK SYMBOLS

ϵ	rate of dissipation of turbulence kinetic energy
$\hat{\epsilon}$	modified rate of dissipation of turbulence kinetic energy, $\epsilon=\hat{\epsilon}$-D
λ	thermal conductivity
μ	dynamic viscosity
μ_t	turbulent dynamic viscosity

ν	kinematic viscosity
Π	pressure term in low-Reynolds-number k-equation
ρ	density
σ_t	turbulent Prandtl number
σ_k, σ_ϵ	turbulent Prandtl number for diffusion of k,ϵ

SUBSCRIPTS

cp	constant property forced convection
vp	variable property forced convection
w	wall

ABBREVIATIONS

ML	Mixing length model [11,12,13]
JL	Jones and Launder low-Reynolds-number k~ϵ model [5,6]
LS	Launder and Sharma low-Reynolds-number k~ϵ model [7]
CH	Chien low-Reynolds-number k~ϵ model [8]
SH	Shih and Hsu low-Reynolds-number k~ϵ model [9]
TAS	Thangam, Abid and Speziale low-Reynolds-number k~ϵ model [10]

1. INTRODUCTION

In recent years, there has been a great concentration of effort in industry on computational modelling of problems involving turbulent fluid flow and heat transfer, usually using codes of considerable versatility. It is sometimes been mistakenly assumed that the turbulence models incorporated in such codes possess more universality than is the case and this can lead to incorrect usage of the codes and wrong conclusions being drawn from the results obtained. Over the years a wide variety of turbulence models have been proposed. Particular attention has been paid to features which enable them to respond in wall shear flows to influences such as severe pressure gradient. A whole family of low Reynolds number k~ϵ models exists incorporating damping functions which aim to describe the turbulence characteristics under conditions where these might be modified by near-wall influences. In the study on which this paper is based [1], a number of such models have been implemented in a computer code, originally produced by Cotton [2], for the computation of buoyancy-influenced developing flow and heat transfer in pipes which was developed later by Yu [3] to include variable property effects. Using these models simulations have been performed with the same solution scheme in each case and the same convergence criteria. Unbiased direct comparisons can therefore be made. All the models cover the whole region of the flow up to the wall without any recourse to using wall functions to bridge the turbulent flow domain and the solid boundary. Results are reported which allow direct comparison to be made between simulations using a variety of models and a selection of experimental data obtained recently by Buyukalaca [4] for buoyancy-opposed turbulent mixed convection in a vertical heated pipe.

The governing equations are written in the 'boundary layer' approximation form and are solved using finite difference approximation and a marching procedure. Five different turbulence closures from the k~ϵ family were examined in this study. The ones selected were: JL (Jones and Launder [5,6]), LS (Launder and Sharma [7]), CH (Chien [8]), SH (Shih and Hsu [9]) and TAS (Thangham, Abid and Speziale [10]). Additionally, a version of the Prandtl mixing length model [11] utilising the Nikuradse distribution of mixing length [12] and the Van Driest [13] wall damping term has been included in the study.

2. GOVERNING EQUATIONS

Since the flow configuration considered here is pipe flow the governing equations are written in the 'boundary layer' approximation with the principal flow direction coinciding with the axis of the pipe. The main gradients are therefore in the direction normal to the axis. The equations are as follows:

Continuity equation

$$\frac{1}{r}\frac{\partial(\rho r V)}{\partial r} + \frac{\partial(\rho W)}{\partial z} = 0 \qquad (1)$$

Momentum equation

$$\frac{1}{r}\frac{\partial(r\rho VW)}{\partial r} + \frac{\partial(\rho W^2)}{\partial z} = -\frac{dp}{dz} + \frac{1}{r}\frac{\partial}{\partial r}\left[r\,(\mu + \mu_t)\frac{\partial W}{\partial r}\right] \pm \rho g \qquad (2)$$

Energy equation

$$\frac{1}{r}\frac{\partial(r\,\rho\, Vh)}{\partial r} + \frac{\partial(\rho\, Wh)}{\partial z} = \frac{Dp}{Dt} + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\lambda}{c_p} + \frac{\mu_t}{\sigma_t}\right)\frac{\partial h}{\partial r}\right] \qquad (3)$$

3. TURBULENCE CLOSURE

In order to solve the above equations the concept of turbulent viscosity is employed. This can be expressed as:

$$\mu_t = C\,\rho u' l' \qquad (4)$$

where u′ and l′ are characteristic turbulent velocity and length scales, respectively. Depending on how the velocity and length scales are specified, the eddy viscosity can be obtained by a variety of models. These can be categorised as: mixing length models, one-equation models or two-equation models.

Two equation models incorporate transport effects on both the velocity scale and length scales which appear in the constitutive equation (4) for turbulent viscosity μ_t. In the case of the k~ϵ models the velocity scale is represented by the square root of the turbulence kinetic energy k and the length scale by combining k and its rate of dissipation ϵ in the manner $k^{3/2}/\epsilon$.

Such models are classified as being either of high Reynolds-number or low Reynolds-number type [14]. In the standard high-Reynolds-number model, the k and ϵ transport equations are solved for the region $y^+ \geq 30$ and analytical, semi-empirical wall functions are employed to bridge the near-wall region. In low-Reynolds-number models, the transport equations are solved over the entire flow domain without recourse to wall functions. The latter type has proved to be successful in the simulation of buoyancy-influenced wall shear flows [15], [16].

In the present work the turbulent viscosity μ_t is defined by the equation:

$$\mu_t = C_\mu f_\mu \frac{\rho k^2}{\hat{\epsilon}} \qquad (5)$$

in which C_μ is a constant and f_μ is a damping function which causes the eddy viscosity near the wall to be reduced. $\hat{\epsilon}=\epsilon - D$ is an isotropic dissipation of turbulence kinetic energy which has been introduced for numerical reasons in order to have $\hat{\epsilon}=0$ at the wall. The equations, which define transport of k and ϵ are as follows:

k-transport

$$\frac{1}{r}\frac{\partial(\rho r V k)}{\partial r} + \frac{\partial(\rho W k)}{\partial z} = \mu_t\left(\frac{\partial W}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] - \rho\hat{\epsilon} + \rho\Pi \qquad (6)$$

ϵ-transport

$$\frac{1}{r}\frac{\partial(\rho r V \hat{\epsilon})}{\partial r} + \frac{\partial(\rho W \hat{\epsilon})}{\partial z} = C_1 f_1 \frac{\hat{\epsilon}}{k}\mu_t\left(\frac{\partial W}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu + \frac{\mu_t}{\sigma_{\hat{\epsilon}}}\right)\frac{\partial \hat{\epsilon}}{\partial r}\right] - C_2 f_2 f_3 \frac{\rho\hat{\epsilon}^2}{k} + \rho E \qquad (7)$$

The various models examined in this study differ only in terms of the functions f_μ, f_1, f_2, f_3 and the terms D and E. Some use as a turbulence time scale k/ϵ and solve an equation for ϵ itself (effectively putting D to zero). In others a time scale is defined by the ratio $k/\hat{\epsilon}$, the value of $\hat{\epsilon}$ being equal to zero at the wall. The functions f_2 and f_3 in the ϵ-equation are usually effective only very close to the wall and are introduced to simulate the change in the decay rate of homogeneous turbulence as the turbulent Reynolds number becomes small. The extra term E, or alternatively the function f_1, is introduced to increase the production of ϵ near the wall. All the models considered are of a form such that when f_μ, f_1, f_2 and f_3 are set to unity, and the terms D and E are set to zero, the standard high-Reynolds number version of the k~ϵ model is obtained.

4. RESULTS

In the computational work reported here attention was focused first on the performance of the various models under consideration when used to describe turbulent flow and heat transfer in a pipe under conditions of forced convection without variable property effects (constant properties). In Table 1 the model predictions are compared with experiment for such conditions. The Nusselt number values appearing opposite 'Correlation' in Table 1 were produced using a modified form of the Petukhov-Kirillov correlation equation [17] fitted to constant property heat transfer result from the recent experimental study [4] obtained by careful extrapolation of data for several values of heat flux to the condition of zero heat flux.

Table 1 Fully developed constant property Nusselt number for Prandtl number of 7 calculated using various models and the modified Petukhov-Kirillov correlation.

MODEL	REYNOLDS NUMBER					
	8000	10000	20000	30000	40000	60000
ML	69.401	83.279	147.88	207.77	264.82	373.40
JL	86.437	105.05	191.83	272.55	349.69	497.02
LS	55.528	68.162	127.34	182.68	235.73	337.42
CH	68.584	83.491	153.70	219.46	282.45	402.89
SH	74.617	89.893	161.27	227.81	291.44	413.12
TAS	71.410	86.869	158.80	225.71	289.57	411.39
CORRELATION	72.544	80.603	146.16	208.36	264.05	372.01

It can be seen that the majority of the model predictions agree with the correlation values within 10%. However, in certain cases there are large discrepancies particularly in the case of the JL model which is clearly not well tuned and over-predicts heat transfer coefficient by as much as 35%.

Next, attention was focused on the influences of the temperature dependence of physical properties. For water, the most important ones are due to variation of density and dynamic viscosity with temperature. The influences of other physical properties are also taken into consideration, but the effects of their variation are much less important. The influences of variable properties can take two distinct forms, one stemming from buoyancy forces which arise as a consequence of non-uniformity of density and the other from both axial and radial variations of the transport properties viscosity and thermal conductivity. The former depends on both power input and flow rate, becoming less important and eventually negligible as flow rate increases. Under such conditions we are left virtually only with the effect of viscosity variation. Two separate influences of variable viscosity can be distinguished. The first one is associated with the axial increase of bulk temperature and the second one is connected with the effects of increase of wall-to-bulk temperature difference. The former arises as a result of the fact that Nusselt number is dependent on Reynolds number which increases as the bulk viscosity falls. The effect of radial viscosity variation under conditions of negligible buoyancy influence is to cause an increase of the heat transfer coefficient. This is due to reduction of viscosity in the near-wall region leading to reduced damping of turbulence and therefore impaired turbulent diffusivity of heat. The effect is usually accounted for using the Sieder and Tate correction [18]. The Sieder and Tate correction involves the applicationn of a viscosity ratio factor to the constant property Nusselt number as follows:

$$Nu_{vp} = Nu_{cp}\left(\frac{\mu_b}{\mu_w}\right)^n \qquad (8)$$

The index n is usually assigned the value 0.14. For water at atmospheric pressure the correction is quite small and only results in the enhancement of heat transfer of up to a maximum extent of about 4%.

The effects of viscosity variation increase with increase of power input and with decrease of flow rate. The results of simulations of experiments for conditions where the influences of buoyancy were small (conditions of high flow rate) are presented here in terms of axial distribution of wall temperature in order of decreasing flow rate at fixed power. Figures 1 to 3 show direct comparisons between the model predictions and experiment. It is apparent that only the LS model manages to reproduce the experimental values closely. The other models all suffer from insufficient tuning of the near-wall turbulence parameters.

In order to quantify the radial variable property effect, the viscosity ratio correction factor, index `n' was calculated for all the model simulations performed for conditions where buoyancy influences were small. This was done for the axial locations z/D = 35, 40, 45, 50, 55 and 60. The results are shown in Figure 4. The various models gave widely differing values of n (varying from about 0.25 up to about 0.77). They are all much larger than the Sieder-Tate value of 0.14. The lowest value was yielded by the CH model and the highest one by the TAS model. The LS model gave an index `n' of about 0.5, similar to that from the ML model. The values do not exhibit any systematic Reynolds number dependence. The slight scatter of points at a particular Reynolds number stems from the fact that some slight influences of buoyancy were present which, although small, show up in comparison with those due to viscosity variation. This is a consequence of performing direct simulations of the experiments in a manner which includes account of buoyancy influences. As the Reynolds number decreases, buoyancy influences show up as slight increases in the values of the index `n' since they lead to an enhancement of heat transfer (see later).

Finally, we turn our attention to conditions where buoyancy-influences are strong. A buoyancy parameter ($B=Gr^*/(Re^{3.425}Pr^{0.8})$ of the kind proposed by Jackson and Hall [19,20]

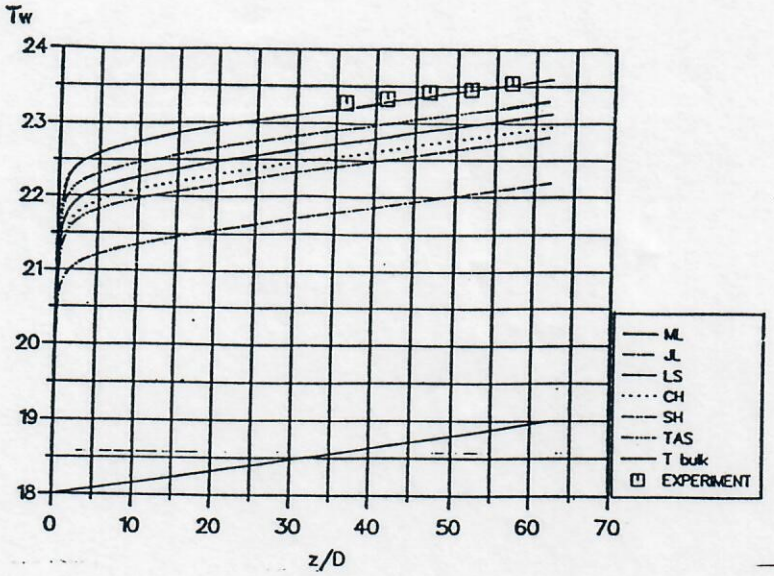


Figure 1 Wall and bulk temperature development;
INLET CONDITIONS: Re=45964, Gr=2.64E8, Pr=7.3

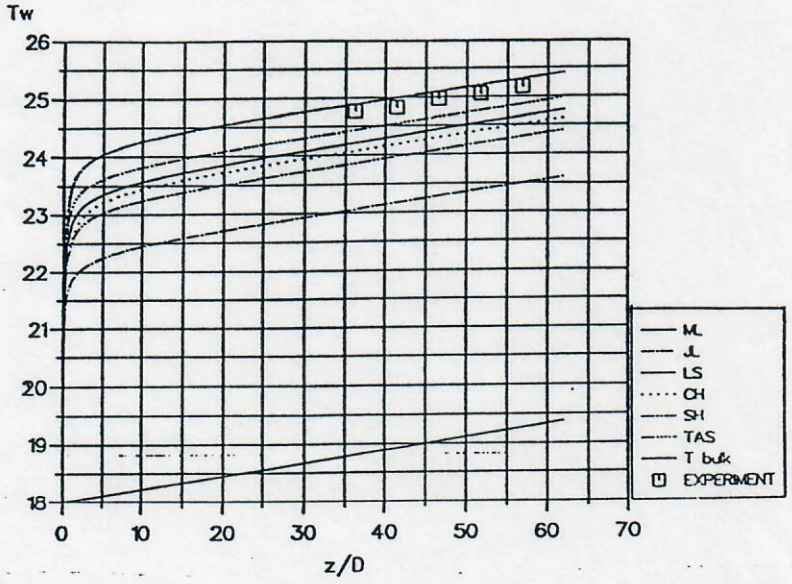


Figure 2 Wall and bulk temperature development;
INLET CONDITIONS: Re=32980, Gr=2.64E8, Pr=7.3

serves to quantify the influence of buoyancy. In the series of experiments simulated in this study the Reynolds number was varied between 14000 and 4500. As a consequence, the buoyancy parameter varied from 3.301×10^{-7} to 1.632×10^{-5}. The results of the simulations are presented in Figures 5 to 8. It is immediately clear from the graphs that again the LS model is the only capable of reproducing the results satisfactorily. These simulations do, however, reveal a small but consistent departure in calculated wall temperature from the experimental values with increase of wall-to-bulk temperature difference. This is the result of the LS model over-responding to viscosity variation. As the flow rate is reduced and the influence of buoyancy becomes stronger the LS model starts to produce some interesting local effects in the first 2-3 diameters (see Fig. 8). These arise as a consequence of the interaction between the effect of thermal development and buoyancy-inpuences as these build up as the thermal layer develops. All the models eventually predict flow reversal and cease to produce results because of the nature of the parabolic computational scheme used in this study.

Buyukalaca [4] achieved an excellent correlation of his buoyancy-influenced heat transfer data in terms of relative heat transfer (Nusselt number for mixed convection normalised by its corresponding forced convection value) plotted as a function of buoyancy parameter. Figure 9 shows the curve which fitted his data along with the prediction of relative heat transfer using the LS model. We see that the numerical results do in fact correlate quite well in terms of relative heat transfer and buoyancy parameter.

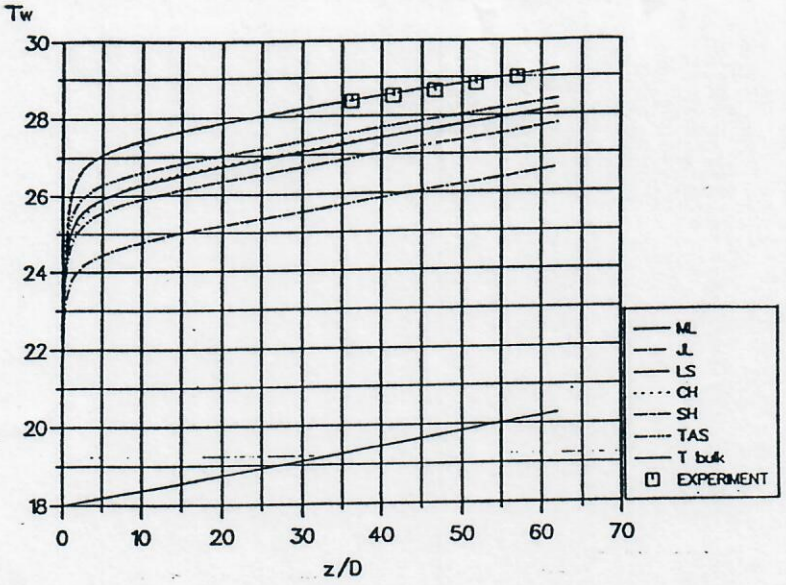


Figure 3 Wall and bulk temperature development
INLET CONDITIONS: Re=19996, Gr=2.64E8, Pr=7.3

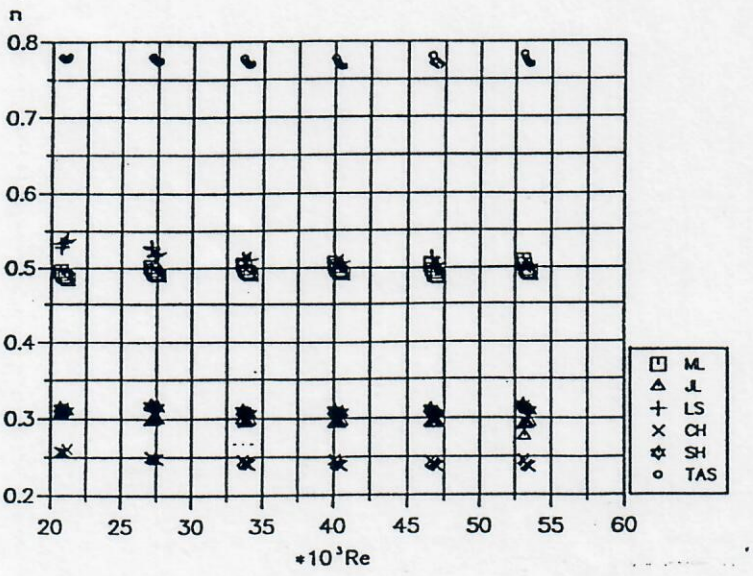


Figure 4 Comparison of the variable property index n for various models.

However, it is apparent that the model predicts buoyancy influences which are somewhat smaller than those found in the experiments. There was no significant dependence of relative heat transfer on axial position. A pseudo-fully developed condition of buoyancy-influenced heat transfer was retained. The JL and CH models gave results showing some enhancement of relative heat transfer but their responces were below that of the LS model. The SH model does not respond to buoyancy influences.

5. CONCLUSIONS

1. For conditions of forced convection with negligible influences of buoyancy the LS model is well tuned to the experimental results. Others, notably the JL model, are not.
2. The effects of viscosity variation are over-predicted by all the models considered here.
3. The LS model clearly performs best in terms of reproducing the influences of buoyancy. The enhancement of heat transfer due to buoyancy is generally underestimated by the models. The SH model does not respond at all.
4. From the present study it seems that the form of parameter in the damping function of the k~∈ turbulence models studied which is best able to respond to buoyancy influences is the local turbulent Reynolds number $Re_t=k^2/v\in$. Formulations, utilizing y^+ in the damping function, under-respond.

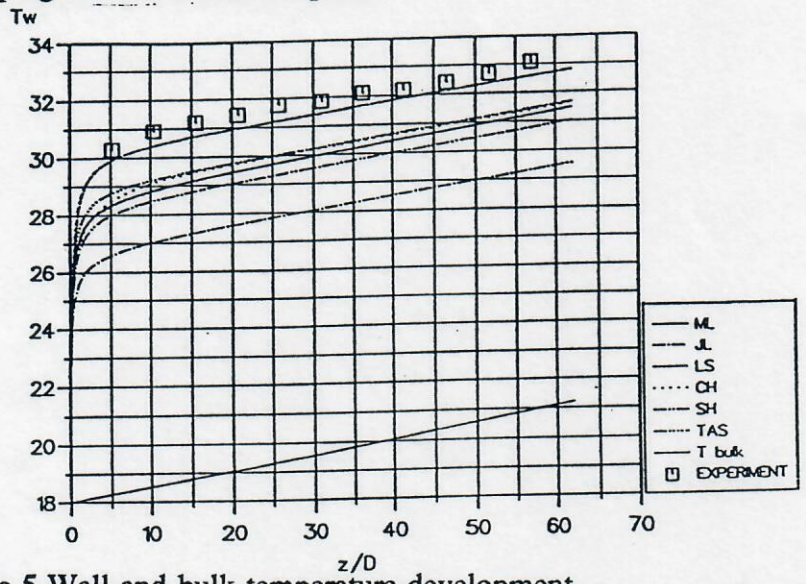


Figure 5 Wall and bulk temperature development
INLET CONDITIONS: Re=14095, Gr=2.64E8, Pr=7.3, B=3.3E-7

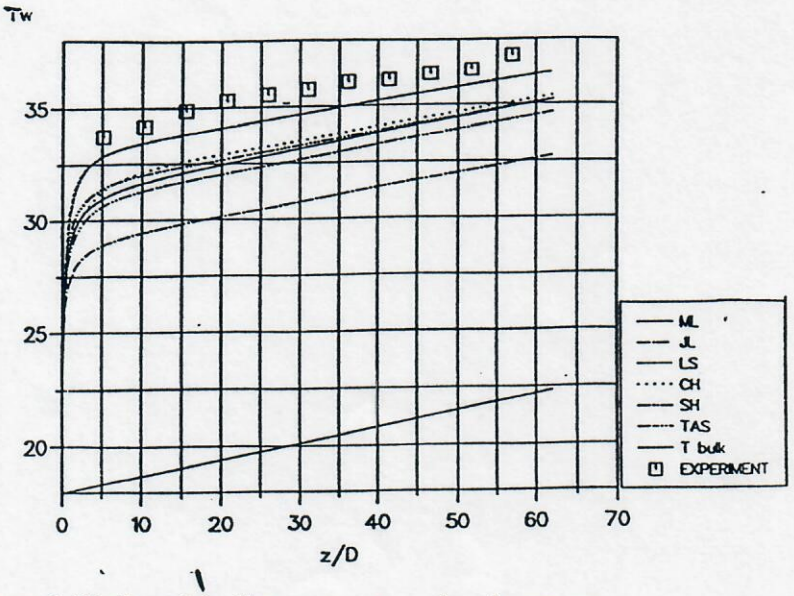


Figure 6 Wall and bulk temperature development
INLET CONDITIONS: Re=10415, Gr=2.64E8, Pr=7.3, B=9.3E-7

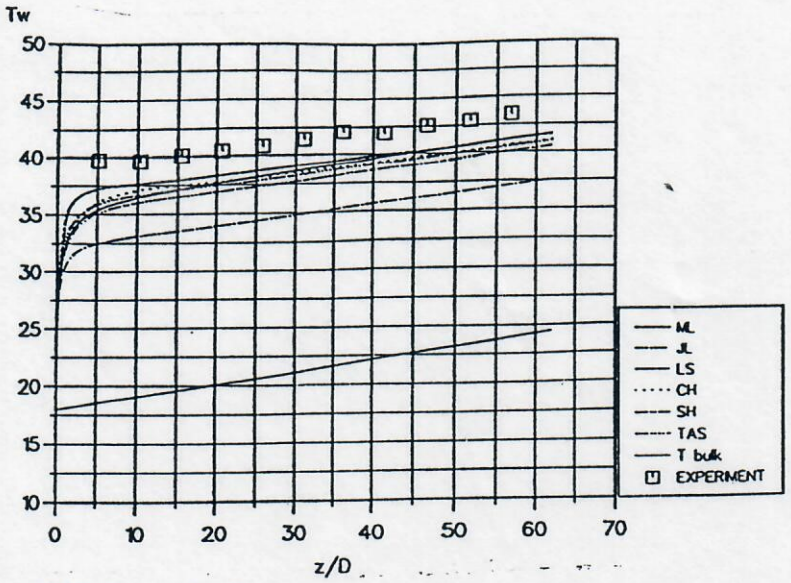


Figure 7 Wall and bulk temperature development
Tw INLET CONDITIONS: Re=7013, Gr=2.64E8, Pr=7.3, B=3.6E-6

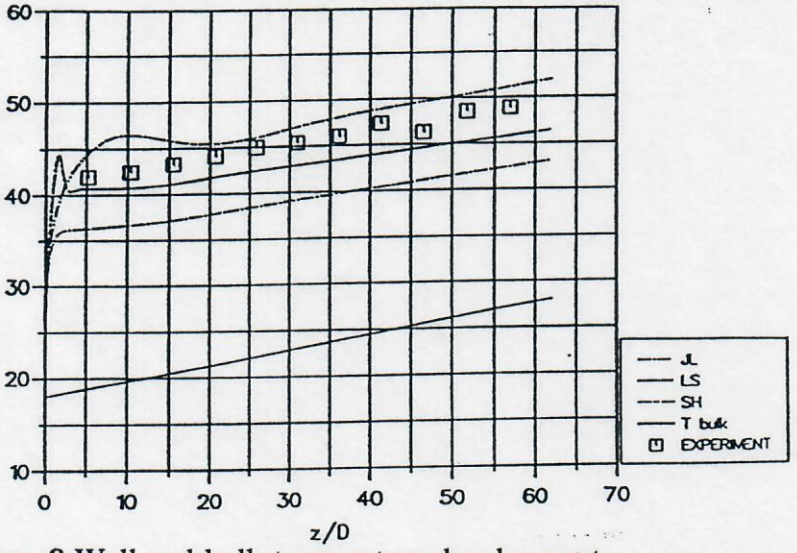


Figure 8 Wall and bulk temperature development
Nu/Nu$_{fc}$ INLET CONDITIONS: Re=4513, Gr=2.64E8, Pr=7.3, B=1.63E-5

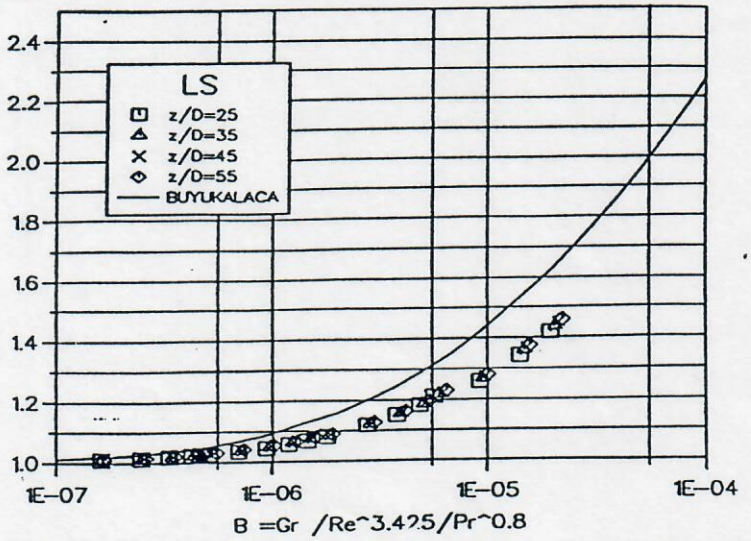


Figure 9 Comparison between Buyukalaca's downward flow correlation and present calculations using the LS model.

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