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Dynamic Analysis of Axisymmetric Shells in the Laplace Domain

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Abstract

In this study, an alternative method for dynamic analysis of axisymmetric shell problems subjected to axisymmetric loading is theoretically investigated. The solutions of damped response of axisymmetric shells have been obtained by finite elements method in conjunction with Laplace transform method using the two dimensional plane elements. Materials of the shells are assumed to be isotropic, linear elastic and/or viscoelastic. In the viscoelastic material case, the Kelvin model is employed. According to the correspondence principle the material constants are replaced with their complex counterparts in the Laplace domain. The governing equations of motion of the problem are first obtained in the time domain. Laplace transform is then applied and the set of simultaneous linear algebraic equations are solved numerically in the Laplace domain. The solutions obtained are transformed to the time domain using the modified Durbin's inverse numerical Laplace transform method. For the suggested model, a finite element analysis computer program is coded in Fortran. Verification of the program is performed by comparing the results of the present method with other numerical methods available in the literature. This procedure has been proved to be highly accurate and efficient compared to various other numerical methods available in the literature.

Keywords: Axisymmetric Shells, Finite Element Method, Inverse Laplace Transform, Kelvin Damping Model

1. INTRODUCTION

There have been several studies on axisymmetric shells, and some of them are, transient response of a thin elastic cylindrical shell to a moving shock wave (Bhuta, 1963), numerical methods in finite element analysis (Bathe & Wilson, 1976), thin shells computing and theory (Gibson, 1984), finite elements for dynamic analysis of shells (Weaver & Johnston, 1984), static and transient response of cylindrical shells (Bhimaraddi, 1987), theory and design of plate and shell structures (Jawad, 1994), thick shells of revolution ; derivation of stiffness matrix of continuous elements and application to a tested cylinder (Casimir et al., 2007) and transient analysis of orthotropic, viscoelastic thick plates in the Laplace domain (Temel & Şahan, 2013).

This paper aims to investigate the dynamic behavior of elastic and viscoelastic shells theoretically in Laplace domain by finite element method. Equations of motion of axisymmetric shells are obtained by Langrangean and Hamilton's principle. The governing equations of motion are obtained in the time domain by using the 8-node quadrilateral finite elements with two degrees of freedom per node. Stiffness matrices, mass matrices and load vectors of the finite elements are computed with three-point Gaussian numerical integration method. The obtained equations of motion of the system are transferred to linear

algebraic equations by Laplace Transform. These simultaneous equations are solved numerically in Laplace domain by Gaussian elimination method for a series of Laplace parameters. Owing to Laplace Transforms the dynamic problems turn into static forms, furthermore the damping effect can be easily addressed in transformed domain. Materials of the shells are assumed to be linear elastic or viscoelastic. The results, obtained in the Laplace domain, are transformed to the time domain with the help of modified Durbin's numerical inverse Laplace transform method (Durbin, 1974, Temel et al.., 2004). For the verification of the presented method, the results are compared with those obtained from ANSYS finite element software which uses the Newmark time integration method to solve the equations of motion.

2. GOVERNING EQUATIONS

Problems involving three-dimensional axisymmetric solids or solids of revolution, subjected to axisymmetric loading, reduce to simple two-dimensional problems. Because of total symmetry about the z-axis, as seen in Fig.1, all deformations and stresses are independent of the rotational angle θ. Thus, the problem needs to be looked at as a two-dimensional problem in *r-z*, defined on the revolving area (Fig. 1b). Gravity forces can be considered if acting in the z direction. Revolving bodies like flywheels can be analyzed by introducing centrifugal forces in the body force term. We now discuss the axisymmetric problem formulation (Chandrupatla, Belegundu, 2002.)

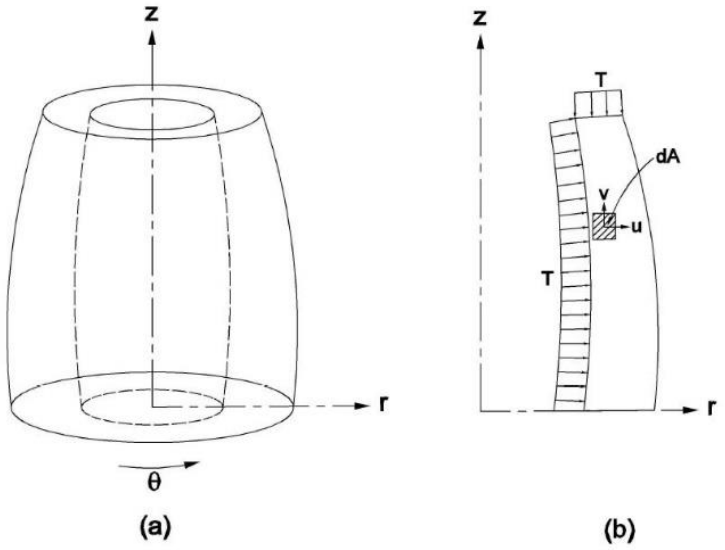


Figure 1: Axisymmetric problem

Displacements of a point are defined as

$$u = u(r, z),\ v = v(r, z) \tag{1}$$

Gravity forces and boundary loads are correspondingly defined as

$$\{f\} = \{f_r, f_z\}^T\ ,\ \{t\} = \{t_r, t_z\}^T \tag{2}$$

we can write the relationship between strains ε and displacements u as

$$\{\varepsilon\} = \{\varepsilon_r, \varepsilon_z, \varepsilon_\theta, \gamma_{rz}\}^T = \left\{\frac{\partial u}{\partial r}, \frac{\partial v}{\partial z}, \frac{u}{r}, \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right\} \tag{3}$$

The stress vector is defined as

$$\{\sigma\} = \{\sigma_r, \sigma_z, \sigma_\theta, \tau_{rz}\}^T \tag{4}$$

Stress-strain relations

$$\begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \tau_{rz} \end{Bmatrix} = \begin{bmatrix} (1-\nu)p & \nu p & \nu p & 0 \\ & (1-\nu)p & \nu p & 0 \\ Symmetry & & (1-\nu)p & 0 \\ & & & G \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix} \tag{5}$$

Where

$$p = \frac{E}{(1+\nu)(1-2\nu)}, \qquad G = \frac{E}{2(1+\nu)} \tag{6}$$

3. FINITE ELEMENT FORMULATION

In this study, 8-node isoparametric quadrilateral finite element has been used. Coordinates of any point of the quadratic quadrilateral element depending on shape functions are denoted as

$$r = \sum_{i=1}^{8} N_i r_i \quad ; \quad z = \sum_{i=1}^{8} N_i z_i \tag{7}$$

Here r_i and z_i are the coordinates of a node, while N_i denotes quadrilateral shape functions. For displacements quadrilateral polynomials with 8 parameters are chosen.

$$u(\xi,\eta) = \{P(\xi,\eta)\}^T\{a\} \quad , \quad v(\xi,\eta) = \{P(\xi,\eta)\}^T\{a\} \tag{8}$$

$\{u_i\}$ displacement for the 8-node reference element can be shown as

$$\{U_d\} = [P_d]\{a\} \tag{9}$$

$$u(\xi,\eta) = \underbrace{\{P(\xi,\eta)\}^T[P_d]^{-1}}_{Shape\ functions}\{U_d\} \tag{10}$$

$$\{N(\xi,\eta)\}^T = \{P(\xi,\eta)\}^T[P_d]^{-1} \tag{11}$$

The Jacobian matrix, which provides the transformation of reference element to real element is as follows

$$[J]=\begin{bmatrix}\frac{\partial r}{\partial \xi} & \frac{\partial z}{\partial \xi}\\ \frac{\partial r}{\partial \eta} & \frac{\partial z}{\partial \eta}\end{bmatrix}=\begin{bmatrix}\sum_{i=1}^{8}\frac{\partial N_i}{\partial \xi}r_i & \sum_{i=1}^{8}\frac{\partial N_i}{\partial \xi}z_i\\ \sum_{i=1}^{8}\frac{\partial N_i}{\partial \eta}r_i & \sum_{i=1}^{8}\frac{\partial N_i}{\partial \eta}z_i\end{bmatrix} \tag{12}$$

$$\begin{bmatrix}\frac{\partial N_i}{\partial r}\\ \frac{\partial N_i}{\partial z}\end{bmatrix}=[J]^{-1}\begin{bmatrix}\frac{\partial N_i}{\partial \xi}\\ \frac{\partial N_i}{\partial \eta}\end{bmatrix}=\underbrace{\frac{1}{det[J]}\begin{bmatrix}\frac{\partial z}{\partial \eta} & -\frac{\partial z}{\partial \xi}\\ -\frac{\partial r}{\partial \eta} & \frac{\partial r}{\partial \xi}\end{bmatrix}}_{[J]^{-1}=[J^*]}\begin{bmatrix}\frac{\partial N_i}{\partial \xi}\\ \frac{\partial N_i}{\partial \eta}\end{bmatrix} \tag{13}$$

The element strain-displacement matrix *[B]* is given by

$$[B_i]=\begin{bmatrix}J_{11}^* & J_{12}^* & 0 & 0 & 0\\ 0 & 0 & J_{21}^* & J_{22}^* & 0\\ 0 & 0 & 0 & 0 & 1/r\\ J_{21}^* & J_{22}^* & J_{11}^* & J_{12}^* & 0\end{bmatrix}\begin{bmatrix}\frac{\partial N_i}{\partial \xi} & 0\\ \frac{\partial N_i}{\partial \eta} & 0\\ 0 & \frac{\partial N_i}{\partial \xi}\\ 0 & \frac{\partial N_i}{\partial \eta}\\ N_i & 0\end{bmatrix} \tag{14}$$

The element stiffness matrix by integration with respect to θ over the domain of *0* to 2π is

$$[k_e]=2\pi\int_{-1}^{+1}\int_{-1}^{+1}[B]^T[C][B]\,r\,det[J]\,d\xi d\eta \tag{15}$$

Gravity forces and boundary load vector of the element joint is calculated as

$$\{f_e\}=\rho\,g\,2\pi\int_{-1}^{+1}\int_{-1}^{+1}\{N\}^T\{f\}\,r\,det[J]\,d\xi d\eta+2\pi\int_{-1}^{+1}\{N\}^T\{T\}\,r\,\frac{L_e}{2}\,d\xi \tag{16}$$

Where g is the standard acceleration due to gravity and ρ is the density of the material.

The element mass matrix by integration with respect to θ over the domain of *0* to 2π is

$$[m_e]=\rho\,2\pi\int_{-1}^{+1}\int_{-1}^{+1}\{N\}^T\{N\}\,r\,Det[J]\,d\xi d\eta \tag{17}$$

Stiffness matrices, mass matrices and load vectors of the finite elements are computed with three-point Gaussian numerical integration method. In this study the equations of motion of axisymmetric shells are obtained by Hamilton's principle. We define the Lagrangean by

$$L = T - \Pi = \frac{1}{2}\{\dot{d}\}^T [M]\{\dot{d}\} - \frac{1}{2}\{d\}^T [K]\{d\} \quad (18)$$

Where *M* is the mass matrix. *K* is the stiffness matrix, $\{\dot{d}\}$ and $\{d\}$ are vectors representing velocity and displacement respectively.

Hamilton's principle - For an arbitrary time interval of *<t1, t2>*, the state of motion of a body extremizes the functional (Chandrupatla, Belegundu, 2002.)

$$I = \int_{t_1}^{t_2} L\, dt \quad (19)$$

L is a function of velocity and displacement thus the equation of the motion is obtained as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{d}_i}\right) - \frac{\partial L}{\partial d_i} = 0 \quad (20)$$

As a result of Eq. (20), the undamped equations of the motion can be obtained as

$$[M]\{\ddot{d}\} + [K]\{d\} = \{F\} \quad (21)$$

4. LAPLACE TRANSFORMS OF THE GOVERNING EQUATIONS

The right side of the Eq. (21), $\{F\}$ is the vector of various time-dependent external forces. The Laplace transform of a time-dependent function *F(t)* is the function *F(s)*.

$$L[f(t)] = \bar{F}(s) = \int_0^{\infty} f(t)\, e^{-st} dt \quad (22)$$

where *s* is the Laplace variable also known as operator variable in the Laplace domain. The Laplace transform of first and second derivatives with respect to time are as follows:

$$L[\dot{f}(t)] = s\bar{F}(s) - f(0) \quad (23)$$

$$L[\ddot{f}(t)] = s^2\bar{F}(s) - s\, f(0) - \dot{f}(0) \quad (24)$$

Application of Laplace transform to Eq. (21) reduces them to

$$(s^2[M] + [K])\{\bar{d}\} - s\,[M]\{d(0)\} - [M]\{\dot{d}(0)\} = \{\bar{F}\} \quad (25)$$

Where $\{\bar{d}\}$ and $\{\bar{F}\}$ are the transformed absolute displacement vector and the transformed external force vector, respectively. While $\{d(0)\}$ is absolute initial displacement and $\{\dot{d}(0)\}$ is absolute initial velocity vectors, which are taken to be zero.

5. EFFECT OF DAMPING

The internal viscoelastic damping case is treated with the aid of the correspondence principle, as described, for example, in Boley and Weiner (1960). The constitutive law of the Kelvin viscoelastic model is

$$S_{ij} = 2G\left(e_{ij} + g\frac{d\,e_{ij}}{dt}\right) \tag{26}$$

where G is the shear modulus and g is the viscous damping coefficient. The deviatoric stress and strain tensors S_{ij} and e_{ij} are defined in terms of the stress and strain tensors s_{ij} and ε_{ij}; respectively, by

$$S_{ij} = \sigma_{ij} - \delta_{ij}\frac{1}{3}\sigma_{kk} \quad , \quad e_{ij} = \varepsilon_{ij} - \delta_{ij}\frac{1}{3}\varepsilon_{kk} \tag{27}$$

Here, δ_{ij} (Kronecker's delta) indicates the components of the identity matrix. The correspondence principle can be stated as follows: the Laplace transform of the viscoelastic solution can be obtained from the Laplace transform of the elastic solution by replacing the elastic constants E and G by

$$E_v = E(1 + gs) \quad , \quad G_v = G(1 + gs) \tag{28}$$

Where E_v and G_v are viscoelastic material constants, and s is the Laplace transform parameter (Boley and Weiner, 1960 and Temel et al., 2004, Şahan, 2012).

6. NUMERICAL EXAMPLES AND DISCUSSION

In this study, a general-purpose computer program is coded in FORTRAN for static and dynamic analysis of axisymmetric shells. The governing equations of motion of the axisymmetric shells are obtained by using FEM. Applying the Laplace transform reduces the dynamic problem to a static one. Thus the obtained simultaneous linear algebraic equations in the Laplace domain are solved numerically by Gauss elimination method for a sequence of values of Laplace parameter. The solutions obtained in the Laplace domain are transformed to the time domain by applying the modified Durbin's inverse numerical Laplace transform method (Durbin, 1974; Narayanan, 1979; Temel, 2003; Pekel et al., 2011).

Displacement and stress values, which are obtained with the presented method are compared with those obtained from ANSYS finite element software, which uses the Newmark time integration method to solve the equations of motion. Comparisons are shown in the graphics.

EXAMPLE 1.

A simply-supported isotropic circular plate, shown in figure 2.a, is now considered under uniformly-distributed dynamic loads. Material and geometrical properties are radius of the plate, r = 1.00 m, thickness, t = 0.10 m, Poisson's ratio, v = 0.3, density, ρ = 7850 kg/m^3 and modulus of elasticity, E = 2 x 10^{11} N/m^2. Two types of dynamic distributed loads, shown in figure 2.b, with the amplitude q_0 = 1 x 10^6 N/m^2 are applied on the plate. The vertical displacements and σ_r stresses at the bottom surface of plate midpoint are obtained by the present model with Laplace transform are compared with those obtained from ANSYS finite element software which uses the Newmark time integration method to solve the equations of motion.

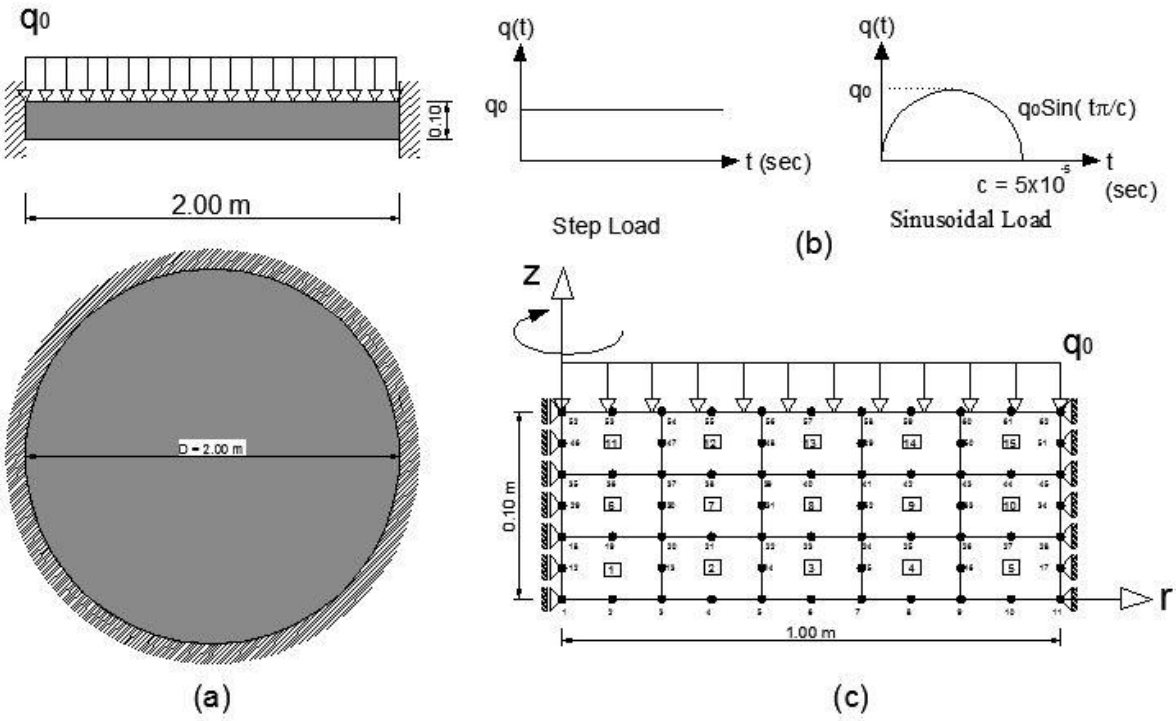


Figure 2: (a) Uniformly loaded circular plate; (b) Dynamic load; and (c) Finite element mesh

The vertical displacement of the plate midpoint under a step load is presented in Fig. 3. The problem has been solved for various time increments as (*dt*=2.0E-6 sec, *N* = 128), (*dt* =1.0E-6 sec, *N* = 256) and (*dt* = 5.0E-5 sec, *N* = 512). It is apparent that results obtained for a coarse time increment along with fewer Laplace transform parameters overlap the results obtained with finer increments and higher parameters. This indicates the efficiency of the present method.

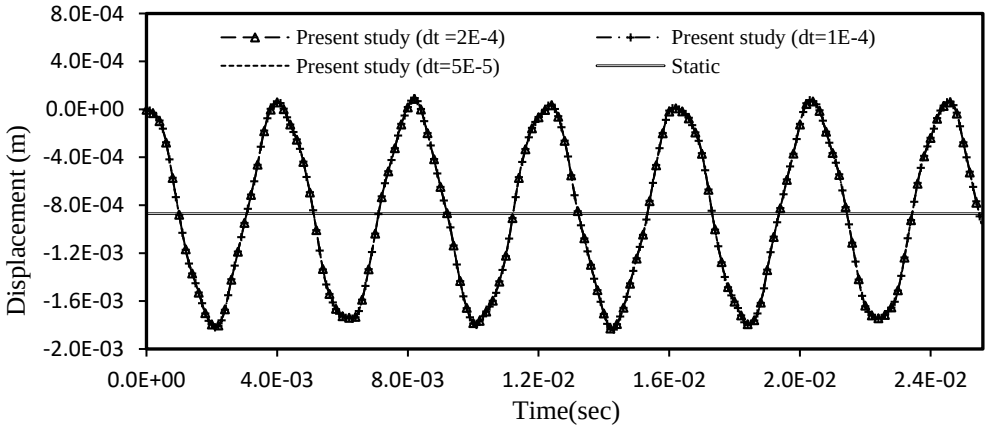


Figure 3: Vertical displacement versus time at the plate midpoint for step load.

Figure 4 shows the vertical displacement results which obtained via the finite element program ANSYS. The comparison of proposed method and Newmark method for various time increments is shown in Figs. 5 and 6 show the displacement and σ_r stresses at the bottom surface of plate midpoint for a step load

respectively. It can be seen that time increments of 5.0E-5 sec and finer had to be considered for consistent results. An exact match is obtained by using a coarse time increment of 2.0E-6 sec in the present model as opposed to much finer increment of 5.0E-5 in Newmark method. The proposed method leads to considerable saving in computation time.

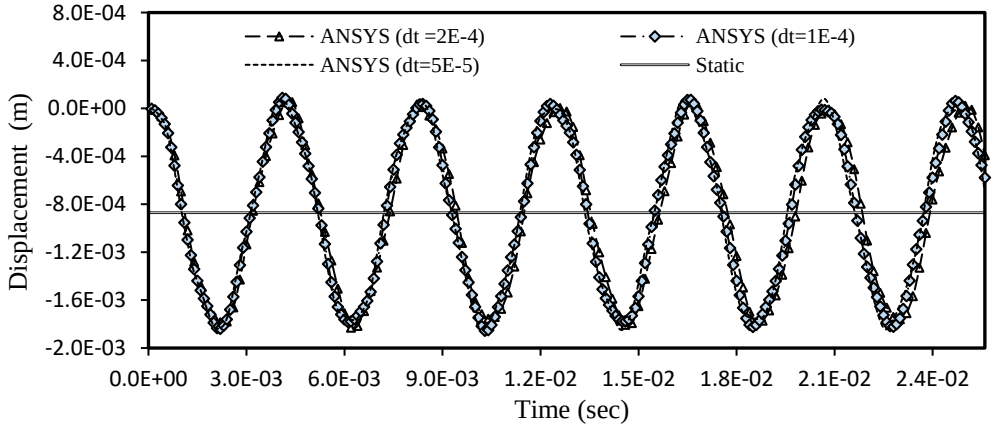


Figure 4: Vertical displacement versus time at the plate midpoint for step load.

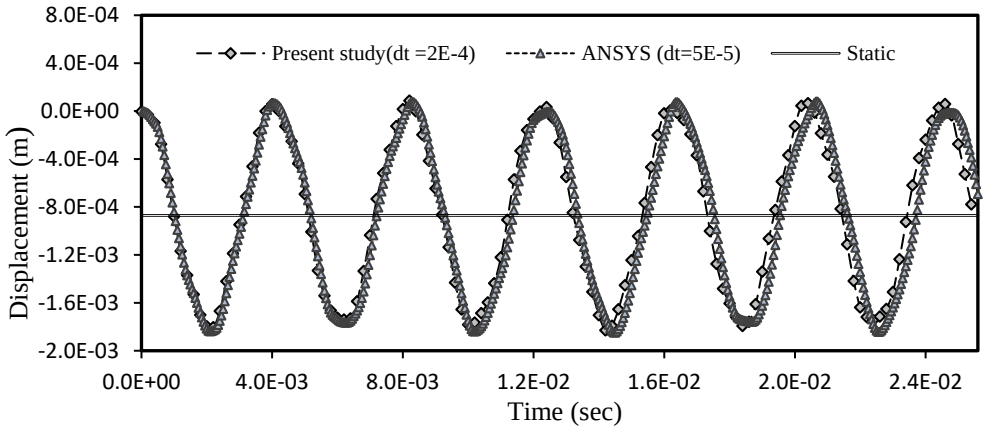


Figure 5: Comparison of vertical displacement versus time at the plate midpoint for step load.

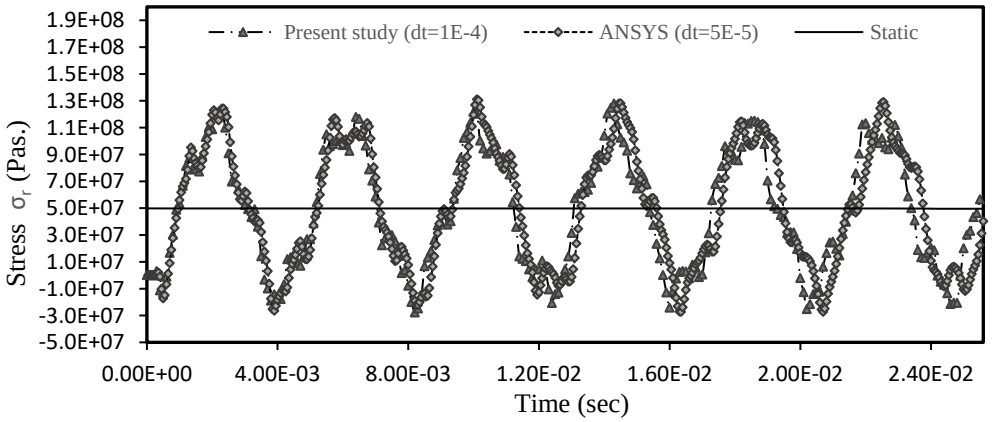


Figure 6: Comparison of σ_r stress versus time at the bottom surface of plate midpoint for step load.

The vertical displacement and σ_r stress of the bottom surface of plate midpoint under a sinusoidal load for viscoelastic material are shown in Figs. 7 and 8, respectively. The figures include static, elastic–dynamic and viscoelastic cases for various damping ratios.

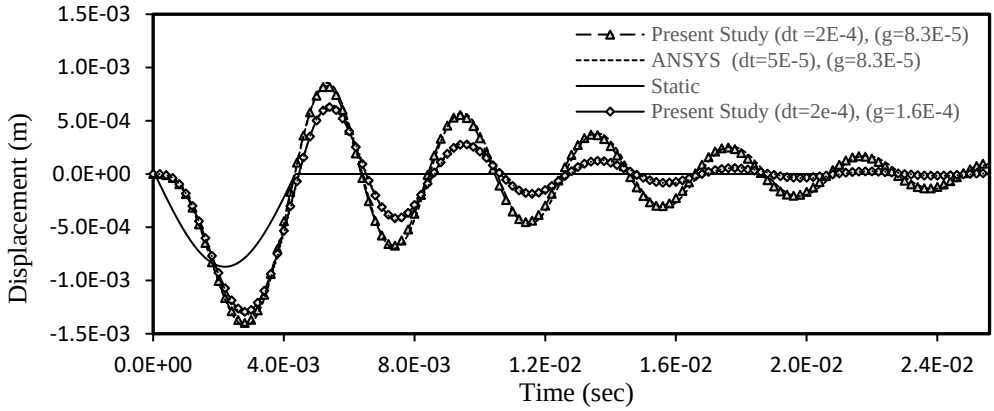


Figure 7: Comparison of vertical displacement versus time at the plate midpoint for sinusoidal load.

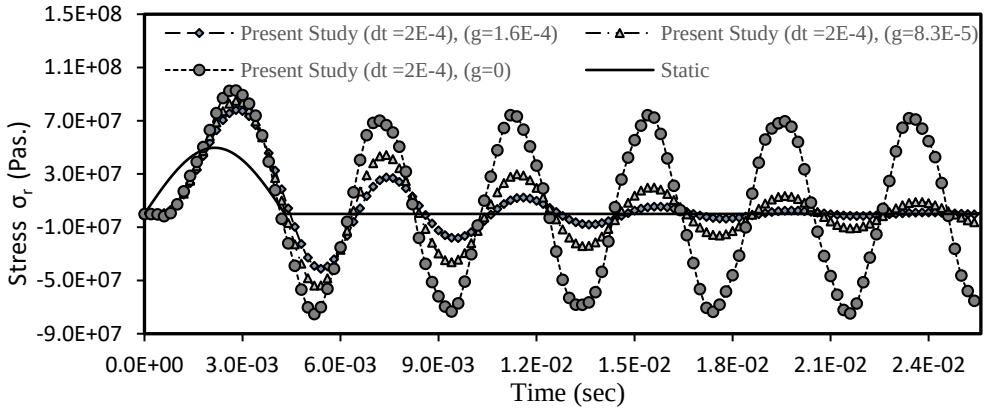


Figure 8: Comparison of σ_r stress versus time at the bottom surface of plate midpoint for sinusoidal load.

EXAMPLE 2.

A simply-supported isotropic dome, shown in figure 9.a, is now considered under a point dynamic load, applied to its midpoint. Material and geometrical properties are radius of the dome, r = 4.50 m, thickness, t = 0.50 m, θ=40°, Poisson's ratio, ν = 0.3, density, ρ = 7850 kg/m^3 and modulus of elasticity, E = 2 x 10^{11} N/m^2. Periodic sinusoidal-wave load, shown in figure 9.b, with the amplitude p_0 = 1 x 10^7 N is applied at the midpoint of the dome. The vertical displacements and σ_r stresses at the bottom surface of dome midpoint are obtained by the present model with Laplace transform are compared with those obtained from ANSYS finite element software which uses the Newmark time integration method to solve the equations of motion.

The comparison of proposed method and Newmark method for various time increments is shown in Figs. 10 and 11 show the displacement and σ_r stresses at the bottom surface of dome midpoint for a periodic sinusoidal-wave load respectively. An exact match is obtained by using a coarse time increment of 2.0E-6 sec in the present model as opposed to much finer increment of 5.0E-5 in Newmark method.

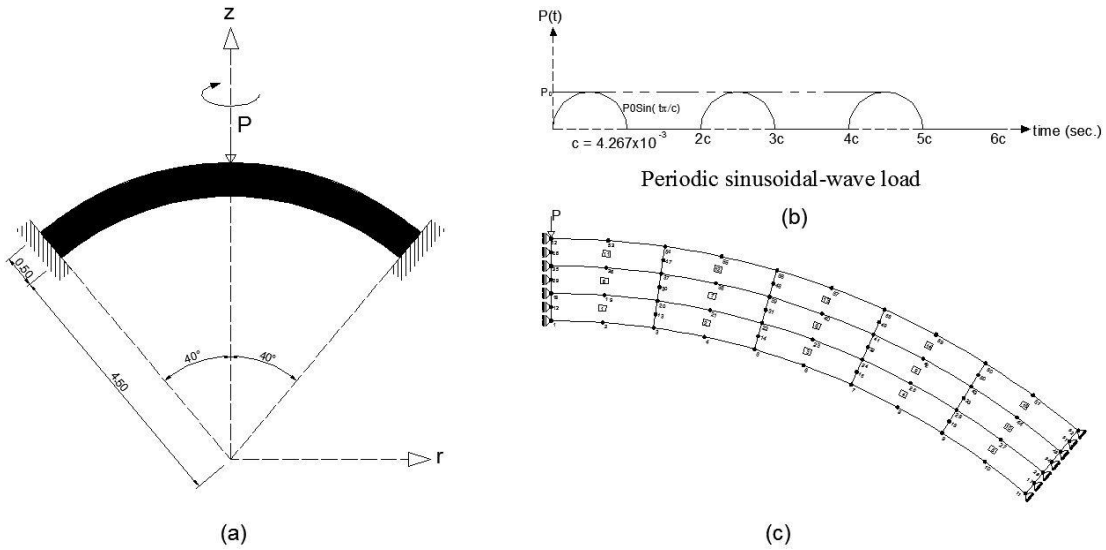


Figure 9: (a) Dome; (b) Dynamic load; and (c) Finite element mesh

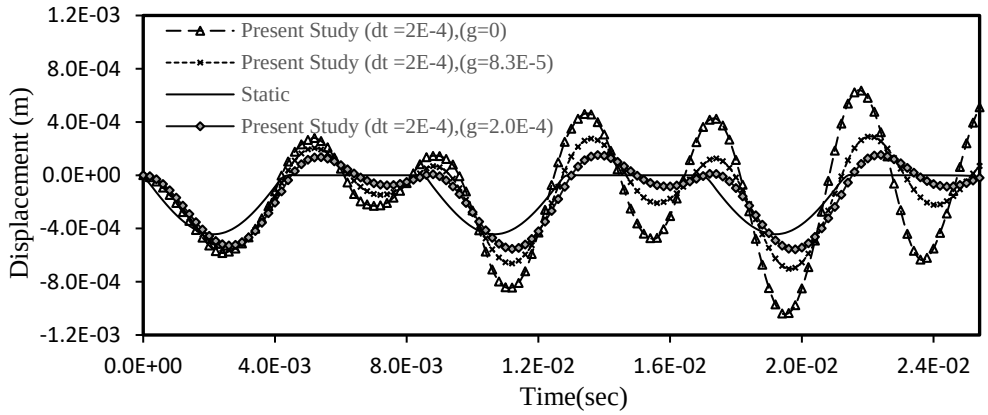


Figure 10: Comparison of vertical displacement versus time at the dome midpoint for a periodic sinusoidal-wave point load at the center of the dome

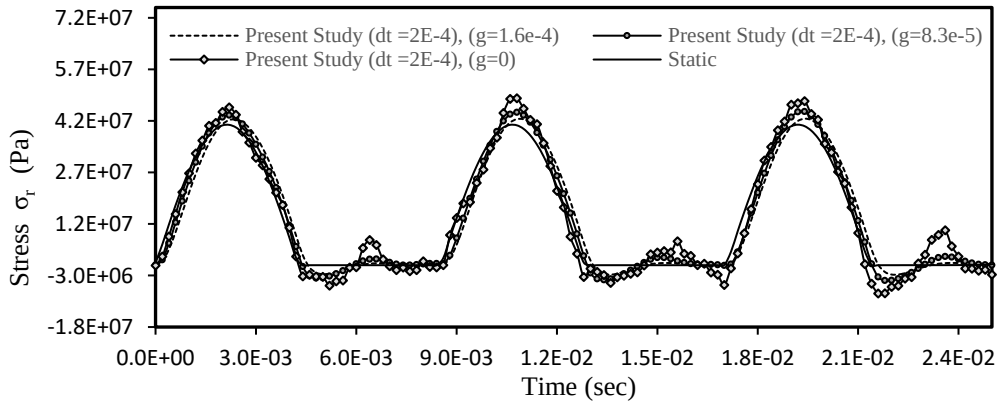


Figure 11: Comparison of σ_r stress versus time at the bottom surface of dome midpoint for a periodic sinusoidal-wave point load at the center of the dome

7. CONCLUSION

In this study, problems involving three-dimensional axisymmetric solids or solids of revolution, subjected to axisymmetric loading, reduce to simple two-dimensional problems. The static analysis and dynamic response of axisymmetric shells made of linear elastic or viscoelastic materials are investigated in the Laplace domain. Owing to Laplace Transform, a dynamic problem is reduced to a static one, which can be solved numerically in the Laplace domain, furthermore the damping effect can be easily incorporated in transformed domain. In the viscoelastic material case, the Kelvin model is employed. According to the correspondence principle the material constants are replaced with their complex counterparts in the Laplace domain.

The accuracy of the results of Newmark method (ANSYS - finite element software) depends on the appropriate selection of the optimum time increment. By using the presented method, highly accurate results can be obtained, even with a coarse time increment. It is manifest that the presented method is far more efficient than the conventional step-by-step integration methods. The numerical examples has proved that that the suggested procedure is highly accurate and efficient compared to various other numerical methods available in the literature and it can be easily applied to the axisymmetric shell problems.

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