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Dynamic magnification response of a vertical plate exposed to breaking wave impact

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Abstract

The dynamic behavior of coastal structures under wave impact closely depends on the temporal and spatial variations of the impact load. The dynamic analyses and design of a coastal structure succeeds if the design load history for the wave impact is available. Since these types of data are very difficult to find, it is more proper to use a method based on static analysis. For this purpose, the ratio of the maximum value of the dynamic response to that found by static analysis is presented as a factor of multiplication and is called "dynamic magnification factor." The dynamic magnification factors from waves breaking directly on vertical coastal plates with built-in edges are calculated for different thicknesses and aspect ratios. The governing equation of the problem is solved by finite element method in which a mesh dependency study is carried out for each plate to find the most appropriate mesh density. From the present computational results, it is suggested that the bending moments from the static analysis of plates can be modified by a factor of 1.15 to obtain the equivalent dynamic bending moments for the plate dimensions in the range of $0.8 \leq a/b \leq 3$ and the wave condition considered in the study.

Keywords

Breaking wave, coastal structure, dynamic magnification factor, impact pressure, vertical plate

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Introduction

The requirements for the constructions of ports and terminals for the receipt of ships and transferring cargos have been increased with the growth of world trade. Such coastal structures are generally protected from waves and high water levels by vertical breakwaters and seawalls. Therefore, the determination of dynamic forces caused by wave breaking on the structure is very important especially for design of seawalls, vertical faced breakwaters, and piers. The experimental data of model and prototype tests on the interaction of coastal structures and waves show that high-magnitude impact (shock) pressures are formed when waves break directly on seawalls which are vertical faced.[1–12] From the past investigations,[13–15] the spatial distribution of impact pressures on the wave–structure interface has become quite well known. There are various dynamic analysis on the spatial and temporal distribution of wave loading on structures using some numerical models;[16–21] however, especially in engineering practice, it is more convenient to follow a method which is based on static analyses.

Using the information obtained from experiments performed on breaking wave impact pressures, the analysis of the response characteristics of a vertical caisson plate of a composite breakwater of prototype

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scale was studied theoretically by Kirkgoz and Mengi[22,23] in which a dynamic magnification factor (DMF) representing the ratio of dynamic and static responses was proposed for a simplified dynamic design procedure of such plates. The past experimental results showed that impact pressures with high-magnitude and very short durations affect the histories of the wall deflection in minor level, and greater wall deflections are produced by the low impact pressures with relatively longer durations.[24] This experimental observation was verified by the results of dynamic analyses carried out by Tanrikulu et al.,[25] in which the experimental wave impact pressures obtained by Kirkgoz[24] was used and the computed vertical wall deflection histories were compared with the measurements, and they concluded that vertical plate of a coastal structure which is exposed to breaking wave impact can be successfully analyzed by employing the classical theory of plates if the spatial and temporal variations of impact pressure values are available. Using the experimental impact pressure histories of Kirkgoz,[24] the static and dynamic analyses of the vertical test plate were carried out by Kirkgoz et al.[26] They found that when $p_{max}/\gamma H_0 \leq 18$ (p_{max} is the maximum impact pressure on the plate, γ is unit fluid weight, and H_0 is deep-water wave height), the maximum static and dynamic wall deflections are agreeably correlated to each other, and the DMF has a range of variation between 0.85 and 1.25 for the wall deflection and span moments.

The design of a coastal plate based on dynamic analysis is not very easy in engineering practice. Therefore, an equivalent design procedure based on the methods of static analysis of the plate is more convenient to follow, using the variation of impact pressures spatially, instead of applying the methods of dynamic analysis directly. In this regard, as a convenient alternative method, the equivalent dynamic analysis of plates used in coastal structures merits further attention for different structural conditions. In this study, using the experimental impact pressure histories from breaking waves, the dynamic behaviors of vertical plates having different dimensions with built-in boundaries were analyzed. Using the spatial distribution of impact pressures, the static analyses of the same plates were also carried out and the DMFs of the plates for deflections and bending moments were investigated.

Experimental data

The experimental data for the impact pressures from waves breaking directly on a vertical wall used in this study were collected by Kirkgoz.[24] In the experiments, a 10-mm-thick vertical steel plate having 2.00×0.57 m dimensions in horizontal and vertical directions was established on a beach with a slope of 1/10 (Figure 1). The details of the experimental setup are given in the mentioned study.[24] Using a U-shaped beam, the upper edge of the plate was strengthened horizontally, while the vertical edges of the plate are free. The lower edge of the plate and the two ends of the beam are assumed to be simply supported ("Hinge" in Figure 1). Ten pressure transducers were framed on the center line of the plate at 30 mm intervals with a start point at 37 mm above the bottom of the plate to measure the spatial distribution of impact pressure histories (Figure 1). A displacement transducer that is fixed to the point 187 mm above the lower edge and 40 mm to the left of center line was used to measure the deflection histories of the wall. The plate is referred to a right-handed Cartesian coordinate system (x, y, z) in which the origin is located at the bottom left corner (Figure 2).

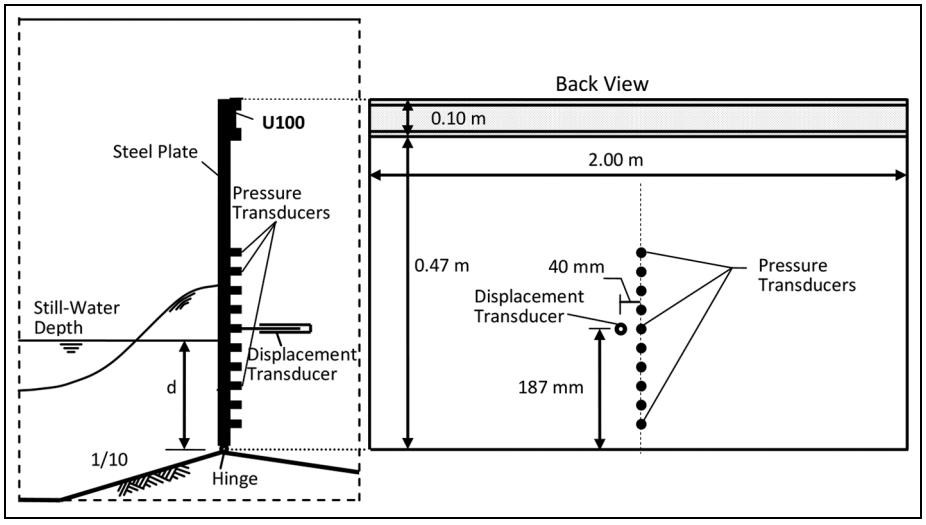

Figure 1. Experimental setup for vertical plate exposed to breaking wave impact.

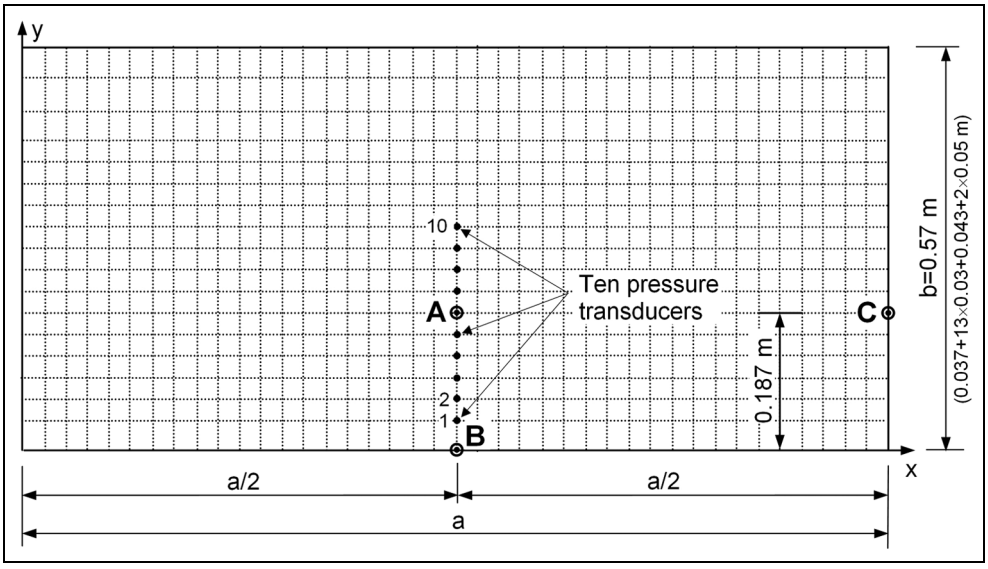

Figure 2. Computational mesh of the plate for finite element analysis.

Regular waves having a period of 2 s, $H_0 = 0.277$ m, and deep-water wave steepness $H_0/L_0 = 0.044$ were used in the experiments (L_0 is the deep-water wave length). The still-water depth at the wall was 0.160 m ("d" in Figure 1). The histories of impact pressure from the 10 transducers were stored at the same time with 25 μs sampling rate for a duration of 102.4 ms, at every impact of 90 waves totally. Figure 3 gives the concurrent vertical pressure distribution at the moment of the highest impact pressure on the plate and the history of the impact force on the plate together with some pressure history examples from the 10 transducers.

Governing equations and numerical solution

The dynamic behavior of the vertical plate exposed to wave impact was formulated using the classical theory of elastic plates.[27] The plate is referred to a right-handed Cartesian coordinate system (x, y, z). The origin is located at the bottom left corner. The (x, y) plane coincides with the mid-plane of the plate (Figure 2). z is in the direction of the force due to the impact pressure $p(x, y, t)$ in which t is time. Figure 3(a) gives the example distributions of pressure in time and space determined experimentally.

The transverse displacement, w, of the plate in z-direction is governed by the following dynamic equation

$$D\nabla^4 w + \rho h\ddot{w} - \frac{\rho h^3}{12}\nabla^2\ddot{w} = p \tag{1}$$

in which $D = Eh^3/12(1-\nu^2)$ is the bending rigidity of the plate, $\nabla^4 = \nabla^2(\nabla^2)$ is biharmonic operator, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is Laplace's operator, ρ is density and h is thickness of the plate, E is modulus of elasticity, ν is Poisson's ratio, and dot represents the differentiation with respect to time. The bending moments in the plate are related to w by

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \tag{2}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \tag{3}$$

where M_x and M_y are the bending moments per unit length which act on the edges of the plate element parallel to the y- and x-axes, respectively.

For the present case, the vertical and horizontal edges of the plate are built-in for which the boundary conditions are

$$(w)_{x=0,a} = 0, \left(\frac{\partial w}{\partial x}\right)_{x=0,a} = 0 \tag{4}$$

$$(w)_{y=0,b} = 0, \left(\frac{\partial w}{\partial y}\right)_{y=0,b} = 0 \tag{5}$$

in which a and b are the width and height of the plate, respectively.

The plate is assumed to start its motion from the rest position. Accordingly, the transverse displacement and velocity of the plate at the initial time $t = 0$ should vanish, which yields

$$(w)_{t=0} = 0, \left(\frac{\partial w}{\partial t}\right)_{t=0} = 0 \quad (6)$$

Plate deflection, w, can be found by solving equation (1), using the boundary and initial conditions defined by equations (4)–(6). The bending moments M_x and M_y can be determined from equations (2) and (3) using the values of w.

The spatial integration in equation (1) was performed using finite element method. Figure 2 shows an example computational mesh employed in this analysis. The mesh spacing in y-direction was so adjusted that the location of each transducer coincided with a nodal point in the mesh. The histories of wall deflection and bending moment were calculated using the histories of impact pressures obtained from the simultaneous recordings of ten transducers. The vertical distribution of impact pressure between the measurement points was assumed to be linear, and it was uniform in horizontal direction. The impact pressures along the bottom edge of the plate were taken equal to the pressure values measured by the nearest transducer (i.e. the transducer 1 in Figure 2). The solution of equation (1) was performed using the software called SAP2000® numerically.[28]

DMF

The static analyses of the plate for w, M_x, and M_y were also performed using the simultaneous vertical impact pressure distributions (an example is given in Figure 3(c)). The theoretical results of the dynamic analyses of the plate are related to the ones obtained from the static analyses using the DMF which was given as a practically usable design tool for the dynamic analysis and the design of coastal plates using the results of static analysis.[22] The DMFs (R) for w, M_x, and M_y are defined as

$$R_w, R_{M_x}, R_{M_y} = \frac{max(w)_{dynamic}}{(w)_{static}}, \frac{max(M_x)_{dynamic}}{(M_x)_{static}}, \frac{max(M_y)_{dynamic}}{(M_y)_{static}} \quad (7)$$

It is stated in Clough and Penzien[29] that the maximum response produced in an undamped single-degree-of-freedom structure by impulsive loading is dependent only on the ratio of the impulse duration to the natural period of the structure, t_1/T_1, and the maximum value of dynamic response ratio (i.e. the DMF) as a function of t_1/T_1 for various forms of impulsive loads was plotted. As given in Figure 3(b), the history of the impact force resembles a triangular impulsive load with duration of t_1. In this study, the variations of the DMFs R_w, R_{Mx}, and R_{My} defined by equation (7) were examined as a function of t_1/T_1 where T_1 is the first natural period of the plate.

Numerical results

The static and dynamic analyses of the vertical steel plates of different aspect ratios and thicknesses, all having built-in boundaries, were carried out using the experimental wave impact pressures. The height of the plate was kept the same for all the cases as $b = 0.57$ m, which was identical with the height of the test plate. Four different widths $a = 0.5$, 1, 2, and 4 m were used for the analyses. The corresponding aspect ratios of the plates are $a/b = 0.88$, 1.75, 3.51, and 7.02. Three different plate thicknesses of $h = 10$, 15, and 20 mm were used for each of the plates. The material properties of the plate are $\rho = 7800\,\text{kg/m}^3$, $E = 2.1 \times 10^{11}\,\text{N/m}^2$, and $\nu = 0.3$. The histories of w, M_x, and M_y at node A, and M_y at node B, and M_x at node C in Figure 2 were computed with 50 μs intervals using the experimental pressure distributions from the impacts of 90 regular waves. The numerical results for the same quantities were also obtained from the static analyses.

Effect of mesh size

Rectangular mesh system used for the solution domain is shown in Figure 2. In order to see the effect of mesh size on the computational results, static analyses of the plates were carried out using different mesh densities. The results for the wall deflection and bending moments of the plates are given in Table 1. The bending moments do not change with plate thickness. As can be seen from the table, the change in the moment values is less than 1%, when the element numbers in x-direction are more than or equal to 40, 40, 80, and 160 for the plates having dimensions of $a = 0.5$, 1, 2, and 4 m, respectively. Having determined the appropriate mesh structure for each case, the corresponding numerical results of the dynamic and static analyses were used to determine the DMFs.

DMFs

The numerical results for the DMFs R defined by equation (7) obtained for the plate of $a/b = 1.75$ with $h = 10$, 15, and 20 mm are given in Figure 4. The scatter plots in the figure show that the values of R do not change with t_1/T_1. The mean value of R in the figure varies in the range of 0.774–1.142, 0.778–1.130, and 0.780–1.105 for $h = 10$, 15, and 20 mm, respectively.

Figure 5 gives the variations of the mean values of R_w, R_{Mx}, and R_{My} for different plate thicknesses with aspect ratio. The figure reveals that the dynamic magnification responses of the plates for different variables behave almost similarly in the sense that they generally

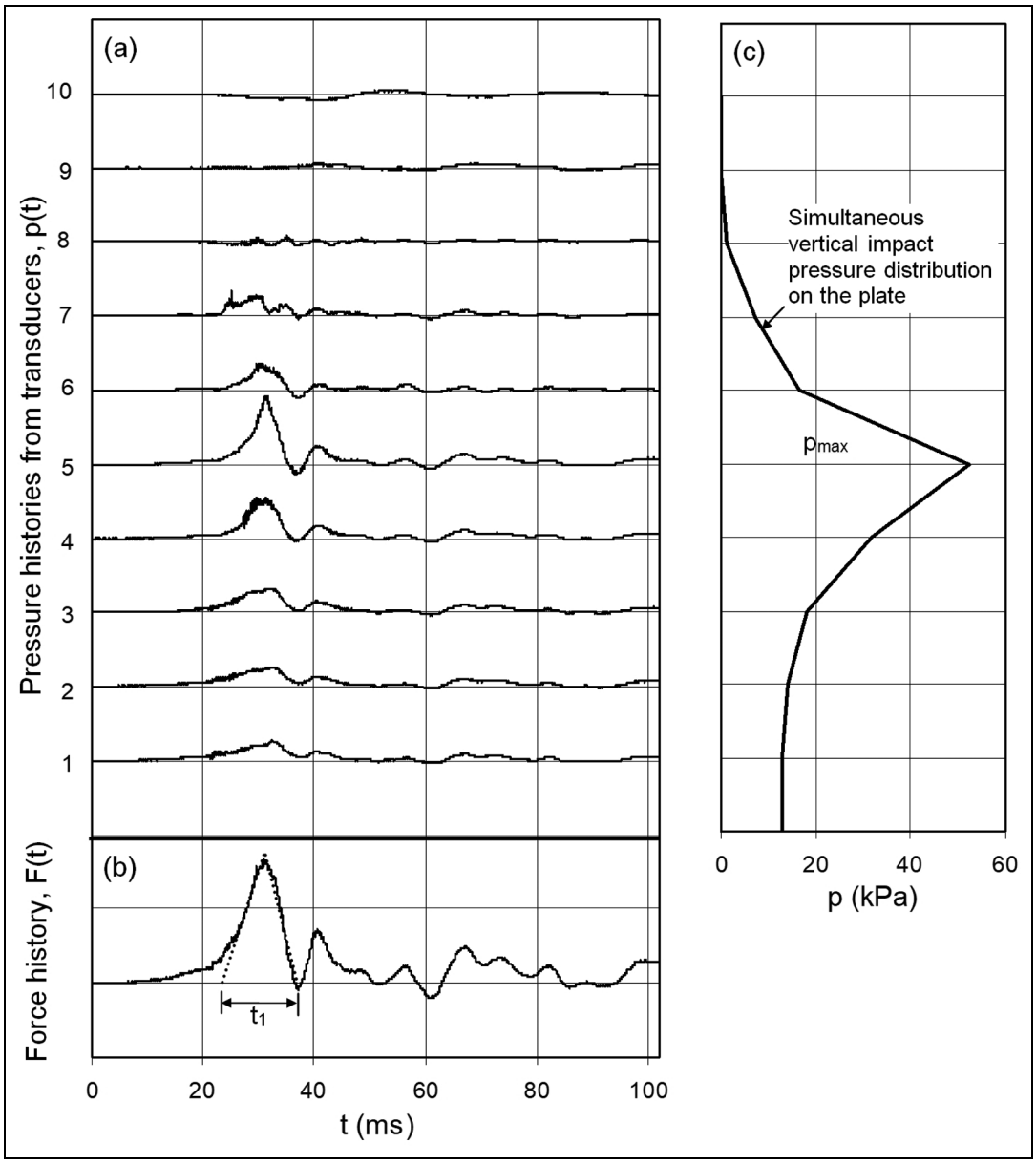

Figure 3. (a) Impact pressure histories from the 10 transducers. The values on the vertical axis (i.e. 1, 2, 3,..., 10) represent the transducer numbers. (b) Impact force history on the plate, and (c) simultaneous vertical impact pressure distribution on the plate at the time when Transducer #5 is at its maximum.

tend to increase to a maximum and then gradually decrease with increasing aspect ratio. It is seen in the figure that the DMFs are not much dependent on the plate thickness; therefore, in each of Figure 5(a)–(e), the average values of factors for the three thicknesses can be used to represent R for a particular a/b.

Variations of the average magnification factors, R, at nodes A, B, and C for the deflection and bending moments with a/b are given in Figure 6. It is seen that for the moments, the highest R values are obtained for M_x at node A between $R_{Mx} = 1.126$ and 0.623, and the lowest values are for M_x at node C between $R_{Mx} = 0.914$ and 0.352 depending on a/b. The present results are quite in agreement with the factors for deflection and span moments found by Kirkgoz et al.[26] for similar aspect ratios. Figure 6 shows that R_{Mx} and R_{My} at A, and R_{My} at B attain their maximums at $a/b = 1.75$ as 1.126, 1.061, and 1.016, respectively, and they are greater than 1.0 in the range of $0.8 \leq a/b \leq 3$; it means that, the dynamic responses of the plate are greater than those of static. Consequently, the rectangular plates having aspect ratios in the range of $0.8 \leq a/b \leq 3$ should be regarded as the most critical conditions for the dynamic magnification response.

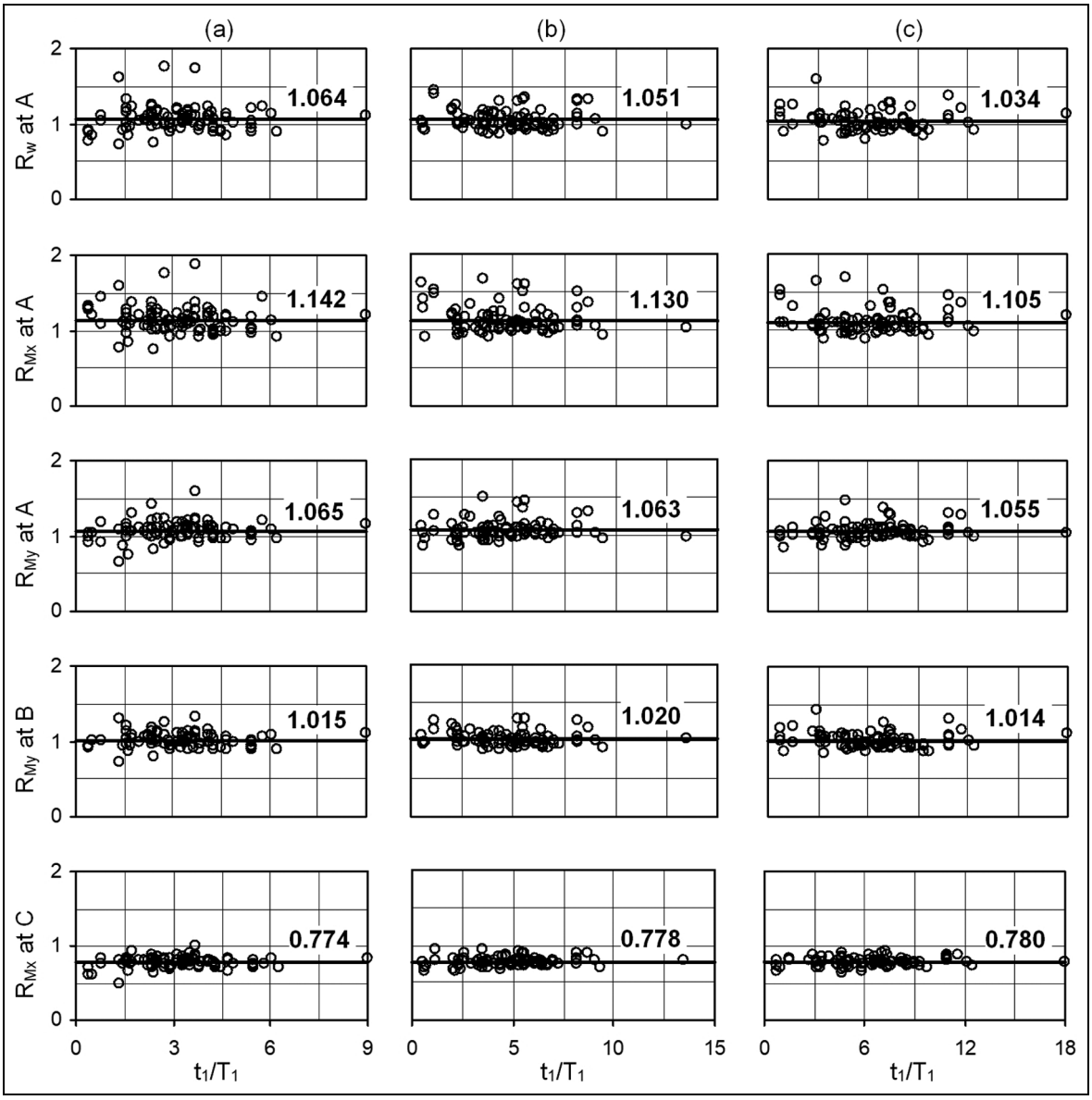

Figure 4. Dynamic magnification factors against t_1/T_1 for the plate of $a/b = 1.75$ with thicknesses of (a) $h = 10$ mm, (b) $h = 15$ mm, and (c) $h = 20$ mm.

The results of DMFs show that using the modified results of static analysis, the dynamic design of coastal plates can easily be achieved without having to confer much more complicated methods of dynamic analysis.

Suggestions for engineering practice

In designing the vertical coastal plates with built-in edges under the breaking wave load, the DMFs given in Figure 6 can be used to modify the results of static analysis to obtain the dynamic calculations of deflection and bending moments. However, it seems more practical to suggest some generally acceptable R values for engineering applications. It was reported that when the aspect ratio $a/b > 3$, the plate can be assumed to behave like an infinitely long plate and the calculations for such a plate can be replaced by those for a strip.[27] Taking into account of this conclusion and the present results for bending moments in Figure 6, the following approximation for the DMF can be suggested for the calculation of dynamic bending moments of plates having aspect ratios in the range of $0.8 \leq a/b \leq 3$, which is the case that mostly encountered in engineering practice

$$R_M = 1.15 \tag{8}$$

R_M in equation (8) represents the DMF for the bending moments. For the design of vertical plates with a/b outside the above range, the static bending moments can

Table 1. Numerical results from static analyses of plates with different mesh densities.

Plate dimensions $a \times b$ (m)	Element numbers ($x \times y$)	Node A					Node B	Node C
		w (mm)			M_x (kN m)	M_y (kN m)	M_y (kN m)	M_x (kN m)
		h = 10 mm	h = 15 mm	h = 20 mm				
0.50 × 0.57	20 × 17	0.081	0.024	0.010	122.31	163.17	343.38	294.77
	40 × 17	0.081	0.024	0.010	121.92	162.68	342.34	296.27
	80 × 17	0.081	0.024	0.010	121.82	162.55	342.09	296.32
	160 × 17	0.081	0.024	0.010	121.80	162.55	342.02	296.28
1.00 × 0.57	20 × 17	0.165	0.049	0.021	88.02	232.73	528.41	313.89
	40 × 17	0.165	0.049	0.021	88.16	232.36	527.48	324.42
	80 × 17	0.165	0.049	0.021	88.19	232.27	527.24	326.20
	160 × 17	0.165	0.049	0.021	88.20	232.25	527.18	326.31
2.00 × 0.57	40 × 17	0.174	0.051	0.022	69.84	233.41	538.10	313.23
	80 × 17	0.174	0.051	0.022	69.86	233.42	538.11	323.77
	160 × 17	0.174	0.051	0.022	69.87	233.42	538.11	325.56
	240 × 17	0.174	0.051	0.022	69.87	233.42	538.11	325.67
4.00 × 0.57	80 × 17	0.173	0.051	0.022	69.96	233.19	537.40	313.23
	160 × 17	0.173	0.051	0.022	69.96	233.19	537.40	323.77
	240 × 17	0.173	0.051	0.022	69.96	233.19	537.40	325.24
	320 × 17	0.173	0.051	0.022	69.96	233.19	537.40	325.56

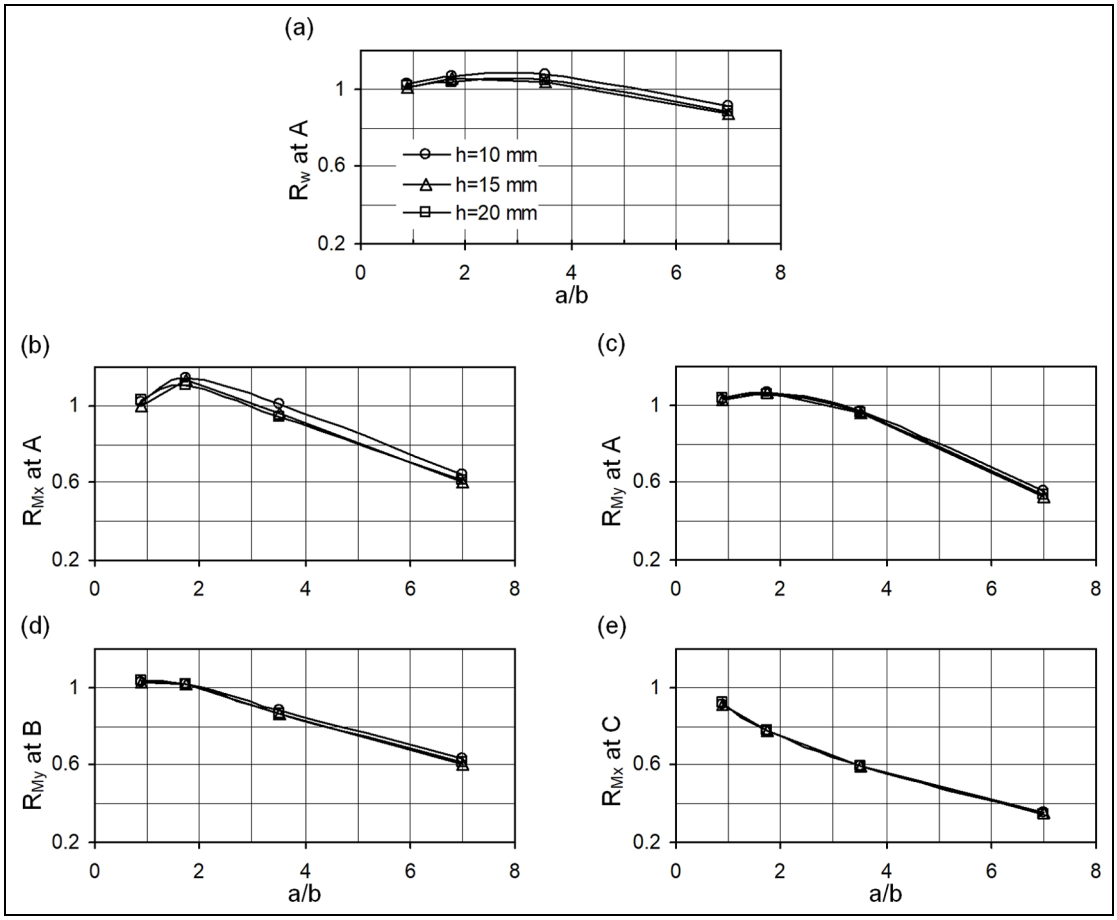

Figure 5. Variations of dynamic magnification factors (a) R_w at node A, (b) R_{Mx} at node A, (c) R_{My} at node A, (d) R_{My} at node B, and (e) R_{Mx} at node C with aspect ratio for plates of different thicknesses.

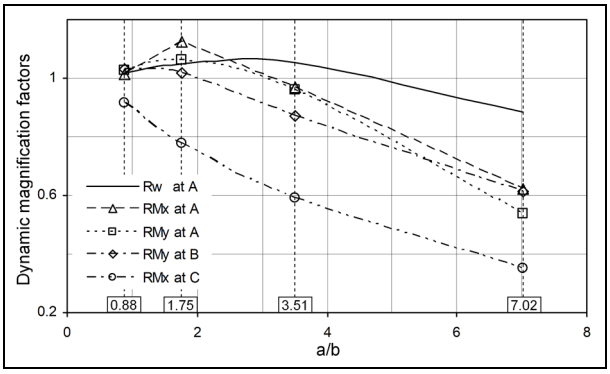

Figure 6. Variations of dynamic magnification factors with aspect ratio.

safely be used by assuming $R_M = 1$. However, as stated by Kirkgoz et al.,[26] one should bear in mind that the present approach for the dynamic design procedure based on the results of static analysis should be limited to the design wave cases in which $p_{\max}/\gamma H_0 \leq 18$. Otherwise, the design calculations may end up with considerable overestimations because $p_{\max}/\gamma H_0 = 18$ indicates the upper limit of the impact load that causes the maximum structural response.[26]

Conclusion

The static and dynamic analyses of vertical steel plates of built-in edges with different thicknesses and aspect ratios were performed using the wave impact pressure data from a model test. Pressure histories from the breaking wave impact were used for the finite element analysis of the dynamic behavior of the plates, and the plate deflection and bending moment histories were calculated. The static analyses of the plates for the deflections and bending moments were also performed using the vertical simultaneous impact pressure distributions on the plate. The ratio of the maximum dynamic and static plate deflections and bending moments was defined as the DMF.

The results show that the DMF is almost independent of the plate thickness and it has a range of variation between 1.126 and 0.352 for the plate aspect ratios between $a/b = 0.88$ and 7.02. For the present computational cases, the plate with $a/b = 1.75$ gives the highest DMFs for most of the calculated deflections and moments. For $a/b > 1.75$, the value of DMF decreases with increasing aspect ratio. From the present numerical results, a value of $R_M = 1.15$ can be suggested for the bending moments of plates having aspect ratios in the range of $0.8 \leq a/b \leq 3$. As mentioned in the previous section, these results are valid for the analyzed wave condition. For plates with a/b outside this range, $R_M = 1$ can be used. In order to take into account of the dynamic response of the coastal plates exposed to wave breaking, the results for the bending moments obtained from the static analysis can be modified by the suggested R_M values. Consequently, by employing the modified results of static analysis, the engineering design of vertical coastal plates under breaking waves can easily be achieved without having to perform more complicated dynamic analysis.

Declaration of conflicting interests

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References

1. Peregrine DH. Water-wave impact on walls. *Annu Rev Fluid Mech* 2003; 35: 23–43.
2. Bullock GN, Obhrai C, Peregrine DH, et al. Violent breaking wave impacts. Part 1: results from large-scale regular wave tests on vertical and sloping walls. *Coast Eng* 2007; 54: 602–617.
3. Ryu Y and Chang KA. Green water void fraction due to breaking wave impinging and overtopping. *Exp Fluids* 2008; 45: 883–898.
4. Bredmose H, Peregrine DH and Bullock GN. Violent breaking wave impacts. Part 2: modelling the effect of air. *J Fluid Mech* 2009; 641: 389–430.
5. Cuomo G, Allsop W, Bruce T, et al. Breaking wave loads at vertical seawalls and breakwaters. *Coast Eng* 2010; 57: 424–439.
6. Cuomo G, Piscopia R and Allsop W. Evaluation of wave impact loads on caisson breakwaters based on joint probability of impact maxima and rise times. *Coast Eng* 2011; 58: 9–27.
7. Ariyarathne K, Chang KA and Mercier R. Green water impact pressure on a three-dimensional model structure. *Exp Fluids* 2012; 53: 1879–1894.
8. Korobkin A. Global characteristics of jet impact. *J Fluid Mech* 1996; 307: 63–84.
9. Korobkin A. Asymptotic theory of liquid–solid impact. *P Roy Soc A: Math Phy* 1997; 35: 507–522.
10. Korobkin A and Khabakhpasheva TI. Plane problem of asymmetrical wave impact on an elastic plate. *J Appl Mech Tech Phy* 1998; 39: 782–791.
11. Carratelli EP, Viccione G and Bovolin V. Free surface flow impact on a vertical wall: a numerical assessment. *Theor Comp Fluid Dyn* 2016; 30: 403–414.

12. Hofland B, Kaminski M and Wolters G. Large scale wave impacts on a vertical wall. *Coast Eng Proc* 2011; 1: 1–15.
13. Mogridge GR and Jamieson WW. Wave impact pressures on composite breakwaters. In: *Proceedings of the 17th conference on coastal engineering*, Sydney, NSW, Australia, 23–28 March 1980, pp.1829–1848. Reston, VA: ASCE.
14. Blackmore PA and Hewson PJ. Experiments on full-scale wave impact pressures. *Coast Eng* 1984; 8: 331–346.
15. Chan ES and Melville WK. Deep-water plunging wave pressures on a vertical plane wall. *P Roy Soc A: Math Phy* 1988; 417: 95–131.
16. Ye JH, Jeng D-S, Chan AHC, et al. 3D integrated numerical model for Fluid-Structures-Seabed Interaction (FSSI): loosely deposited seabed foundation. *Soil Dyn Earthq Eng* 2017; 92: 239–252.
17. Ye JH, Jeng D-S, Chan AHC, et al. 3D integrated numerical model for Fluid-Structures-Seabed Interaction (FSSI): elastic dense seabed foundation. *Ocean Eng* 2016; 115: 107–122.
18. Ye JH, Jeng D-S, Wang R, et al. Numerical simulation of wave-induced dynamic response of poro-elastoplastic seabed foundations and a composite breakwater. *Appl Math Model* 2015; 39: 322–347.
19. Ye JH, Jeng D-S, Wang R, et al. Breaking wave-induced response of composite breakwater and liquefaction in seabed foundation. *Coast Eng* 2014; 85: 72–86.
20. Ye JH, Jeng D-S, Wang R, et al. Validation of a 2D semi-coupled numerical model for fluid-structures-seabed interaction. *J Fluid Struct* 2013; 42: 333–357.
21. Jeng D-S, Ye JH, Zhang J-S, et al. An integrated model for the wave-induced seabed response around marine structures: model verifications and applications. *Coast Eng* 2013; 72: 1–19.
22. Kirkgoz MS and Mengi Y. Dynamic response of a caisson plate to wave impact. *J Waterw Port C: ASCE* 1986; 112: 284–297.
23. Kirkgoz MS and Mengi Y. Design of a caisson plate under wave impact. *Ocean Eng* 1987; 14: 275–283.
24. Kirkgoz MS. An experimental investigation of a vertical wall response to breaking wave impact. *Ocean Eng* 1990; 17: 379–391.
25. Tanrikulu AK, Kirkgoz MS and Dundar C. Theoretical and experimental investigation of a vertical wall response to wave impact. *Ocean Eng* 2002; 29: 769–782.
26. Kirkgoz MS, Tanrikulu AK and Dundar C. Dynamic analysis of a vertical plate exposed to breaking wave impact. *Ocean Eng* 2004; 31: 1623–1635.
27. Timoshenko SP and Woinowsky-Krieger S. *Theory of plates and shells*. New York: McGraw-Hill, 1959.
28. Computers and Structures, Inc. *SAP2000®: integrated finite element analysis and design of structures*. Berkeley, CA: Computers and Structures, Inc., 1999.
29. Clough RW and Penzien J. *Dynamics of structures*. New York: McGraw-Hill, 1993.