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CALCULATION OF ROTARY HEAT EXCHANGERS WITH VARIOUS MATRIX GEOMETRIES

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Abstract

Rotary regenerators are one of the most used and most efficient energy recovery devices. In this study, equations have been developed for the calculation of pressure drop and heat transfer in rotary regenerators. Arbitrary shaped matrix, revolution and cleaning region of regenerator wheel have been considered in the calculation procedure. With the equations given, economical and thermodynamical optimization of the regenerators are possible.

INTRODUCTION

Rotary regenerators have certain advantages over ordinary heat exchangers for recovery of heat from exhaust air. Two main advantages are that an extremely compact and relatively inexpensive heat transfer surface can be used and that the speed of rotation of the matrix can be used to regulate the amount of heat transferred. Thus, rotary regenerators combine both compactness and high performance.

In this work, a design method for rotary regenerators has been developed. With this method, pressure loss and heat transfer can be calculated for various shaped matrix in rotary wheel. In addition, the influence of rotational speed and cleaning region of the regenerator on the effectiveness can also be considered.

CALCULATION OF PRESSURE DROP

The total pressure loss in rotary regenerators can be given as follows:

$$\Delta P = \Delta P_i + \Delta P_r - \Delta P_o \tag{1}$$

where

ΔP_i = Pressure drop at the inlet of the regenerator

ΔP_r = Pressure drop in the regenerator

ΔP_o = Pressure gain at the outlet of the regenerator.

The pressure drop at the inlet of channels is given by the following equation:

$$\Delta P_i = 1.4\left(1-\sigma^2\right)\frac{\rho u_o^2}{2} \tag{2}$$

Here, u_o is the average velocity at the inlet (Fig. 1), ρ the density of the fluid and σ the porosity.

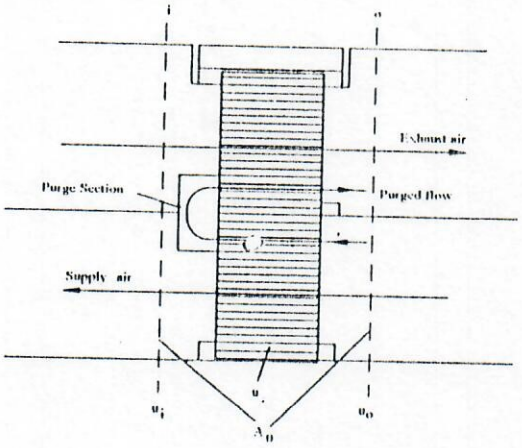


Fig 1. Section of the rotary regenerator

The velocity u_o can be calculated from:

$$u_o = \frac{\dot{M}}{\rho A_o} \tag{3}$$

In this equation A_O is the cross-sectional area of the channel at the inlet.

Using the expression below we can determine the pressure gain at the outlet of the regenerator can be determined :

$$\Delta P_o = 2(1-\sigma)\frac{\rho u_o^2}{2} \tag{4}$$

In the equations given above index i refers to inlet and o to outlet.

Porosity, σ, is defined as below:

$$\sigma = \frac{V - V_s}{V} \tag{5}$$

where, V is the total volume of the heat wheel and V_s the volume of solid material.

Pressure drop in the regenerator can be calculated using the equation given by Yılmaz (1990) :

$$\Delta P = \Delta P^* \frac{\rho u^2}{2} \tag{6}$$

$$u = \frac{\dot{M}}{\rho A_o \sigma} \tag{7}$$

where, u is the actual velocity in the regenerator and ΔP^* is

$$\Delta P^* = 64\psi x^* + \frac{13.766 x^{*0.5}}{\left[1 + 13.95\psi x^{*0.5} + \left(\frac{13.766}{K}\right)^2 x^{*1.5}\right]^{1/3}} \tag{8}$$

In this equation the first term indicates the pressure loss for fully developed velocity profile and second term the incremental pressure loss in the developing velocity profile region. x^* is the dimensionless entrance length and defined as follows:

$$x^* = \frac{x}{d_e} \frac{1}{Re} \tag{9}$$

Definition of Reynolds number and equivalent diameter are given below:

$$Re = \frac{u d_e}{\upsilon} \tag{10}$$

$$d_e = \frac{4A}{P} \tag{11}$$

where A is the cross-sectional area and P the periphery of a channel in the matrix. ψ is the shape factor and can be calculated from the equation:

$$\psi = 1 + \frac{\psi_\infty - 1}{\left[1 + 0.33 d^{*2.5}/(n-1)\right]} \tag{12}$$

where ψ_∞, d^* and n are defined as follows:

$$\psi_\infty = \frac{3}{8} d^{*2}\left(3 - d^*\right) \tag{13}$$

$$d^* = \frac{d_e}{d_{max}} \tag{14}$$

$$n = \frac{P}{P_e} = \frac{A}{A_e} \tag{15}$$

In these equations P_e and A_e are, respectively, the periphery and cross-sectional area of the equivalent diameter and d_{max} is the largest circle suiting the cross-section of the matrix channel. d_{max} is shown in Fig. 2.

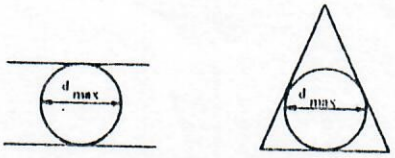


Fig 2. Explanation of maximum diameter

n and d^* describe together the shape of the channel. The equation for the calculation of the incremental pressure drop number K, is given below:

$$K = 1.33 \Big/ \left\{ 1 + (1.33/K_\infty - 1) \Big/ \left[1 + 0.74 d^{*2} / (n-1) \right] \right\} \tag{16}$$

where,

$$K_\infty = \frac{12}{5}\left(3 - d^*\right)^2 \left[\frac{9}{7} \frac{3 - d^*}{7 - 3d^*} - \frac{1}{5 - 2d^*} \right] \tag{17}$$

With the equations given above, one is able to determine the pressure loss in the regenerator.

THERMAL ANALYSIS OF ROTARY REGENERATORS

To calculate the efficiency of a rotary heat exchanger and to design it, heat transfer coefficient α is to be known. In laminar flow, definition of an equivalent diameter is not adequate to calculate heat transfer as in turbulent flow. Some additional quantities must be introduced to describe heat transfer. Heat transfer coefficient is determined by Nusselt number (Nu), which is defined as follows:

$$\alpha = Nu \frac{\lambda}{d_e} \tag{18}$$

For ducts of arbitrary cross-sections and thermally developing flow, Nusselt number can be calculated using the equations given by Yılmaz and Cihan (1993) :

$$Nu = Nu_\infty \left[1 + \frac{4.212 \psi \Phi'}{z^* Nu_\infty'} - 0.8 \left(\frac{\psi \Phi'}{z^* Nu_\infty'} \right)^{2/3} \right]^{1/4} \tag{19}$$

Here Nu_∞ is the Nusselt number for $z^* \to \infty$ and determined from the expression below:

$$Nu_\infty = 3.657 \phi \tag{20}$$

where,

$$\phi = 1 + \frac{\phi_\infty - 1}{1 + 1/(n-1)} + \Delta\phi' \tag{21}$$

$$\phi_{\infty} = 0.5155\frac{d^{*2}}{3-d^{*}} \tag{22}$$

$$\Delta\phi = \Delta\phi_{max}\frac{0.95(n-1)^{0.5}}{1+0.038(n-1)^{3}} \tag{23}$$

$$\Delta\phi_{max} = \frac{7.10^{-4}\,d^{*4}}{\left(1+10d^{*2.8}\right)\left(1+64.10^{-8}d^{*7.8}\right)^{1/2}} \tag{24}$$

The term Φ in Eq (19) is determined by the following equation:

$$\Phi = 1+\frac{\left[3\left(d^{*}/2\right)^{7/8}/\left(1+d^{*}\right)\right]-1}{1+0.25/(n-1)} \tag{25}$$

z^{*}, which is another dimensionless number, is given by

$$z^{*} = \frac{L}{d_e}\frac{1}{Re\,Pr} \tag{26}$$

where Pr is Prandtl number.

Using all the above-given equations together, Nusselt number is calculated and from the calculated Nusselt number, heat transfer coefficient is determined

As Nusselt number is known, it is possible to calculate regenerator efficiency as follows (Yilmaz, 1995)

$$\varepsilon = f\left(N, C^{*}\right) \tag{27}$$

Regenerator effectiveness is defined as

$$\varepsilon = \frac{Q}{Q_{max}} \tag{28}$$

Here, Q indicates the heat transferred and Q_{max} the maximum heat that can be transferred in an ideal heat exchanger

$$Q = C_1(T_{1i}-T_{1o}) = C_2(T_{2o}-T_{2i}) \tag{29}$$

where 1 refers to supply air and 2 to exhaust air.

The maximum possible heat that can be transferred in the regenerator is given by:

$$Q_{max} = C_{min}(T_{1i}-T_{2i}) \tag{30}$$

Here C_1 and C_2 are the thermal capacities and defined as follows.

$$C_1 = \dot{M}_1 c_{p1} \tag{31}$$

$$C_2 = \dot{M}_2 c_{p2} \tag{32}$$

Here, $\dot{M}$ is mass flow rate and c_p specific heat capacity of the fluid. C_{min} is the minimum thermal capacity.

In a counter-current flow heat exchanger, efficiency is calculated by the following equation (Yılmaz, 1995):

$$\varepsilon_o = \frac{1 - e^{\left[-N(1-C^*)\right]}}{1 - C^* e^{\left[-N(1-C^*)\right]}} \tag{33}$$

In this equation C^* and N is defined as;

$$C^* = C_{min}/C_{max} \tag{34}$$

$$N = kF/C_{min} \tag{35}$$

kF is determined as follows:

$$\frac{1}{kF} = \frac{1}{\alpha_1 F_1} + \frac{1}{\alpha_2 F_2} \tag{36}$$

α_1 and α_2 are, respectively, the heat transfer coefficients between the hot and cold fluid and the surfaces of matrix material. They can easily be calculated from the Nusselt numbers given above. F_1 and F_2 are the surfaces which hot and cold fluids contact with and they are usually equal to each other.

For a rotary regenerator, efficiency is calculated using the equation given below.

$$\varepsilon = \varepsilon_o \varphi_r \varphi_c \tag{37}$$

Here, φ_r is a correction factor for the revolution of the wheel and determined as follows (Kays ve London, 1984):

$$\varphi_r = 1 - \frac{1}{9C_r^*} \tag{38}$$

and the capacity ratio, C_r^*, is defined as:

$$C_r^* = \frac{C_r}{C_{min}} \tag{39}$$

where C_r is the thermal capacity of the solid material :

$$C_r = M_s c_{ps} n \tag{40}$$

M_s and c_{ps} are, respectively, the mass and the specific heat capacity of the solid material in the regenerator and n the rotational speed of the regenerator. The correction factor φ_r can also be calculated using the equation given by Worsφe-Schmidt (1991) which is more precise than Eq (38) :

$$\varphi_r = 1 - \frac{0.1\left[1 - e^{(-N)}\right]}{C^{*0.11} C_r^{*1.03}} \tag{41}$$

A factor φ_c is introduced in order to take into account of the influence on effectiveness of the cleaning region. The below equation for factor φ_c has been given by Worsφe-Schmidt (1991)

$$\varphi_c = (1 - a)e^{(-ba)} \tag{42}$$

where,

$$a = M_{lp}/M_l \tag{43}$$

$$b = 2.45 - 0.4C^* - \frac{0.69}{C_r^* C^{*4.3}} \tag{44}$$

M_l and M_{lp} indicate the mass flow rate of fresh air and of the fresh air turning back from the cleaning region respectively.

Using the equations given above, it is possible to design rotary regenerator with the given geometric values.

CONCLUSIONS

The pressure drop and heat transfer in rotary regenerators can be calculated with the equations given in this work. This can be done for all kind of matrix form in regenerators. The revolution and the cleaning region of the regenerator wheel have been taken into consideration. The equations given can be used for economical and thermodynamical optimization of regenerators.

NOMENCLATURE

A	=	cross-sectional area
d_e	=	equivalent diameter
d_{max}	=	maximum diameter of circle suiting actual cross-section
h	=	heat transfer coefficient
k	=	thermal conductivity
L	=	length of the duct
K	=	incremental pressure drop number
Nu	=	Nusselt number
P	=	periphery of cross-section
Δp	=	pressure drop
Δp*	=	dimensionless pressure drop
Re	=	Reynolds number
Pr	=	Prandtl number
Q	=	heat transfer rate
Q_{max}	=	maximum heat transfer rate
T	=	temperature
u	=	velocity
x	=	length of the duct
x*	=	dimensionless length of the duct
z*	=	dimensionless coordinate

Greek Symbols

ρ = density
φ = shape factor for developed flow
Φ = Heat transfer factor, Eq. (25)
ϕ = Heat transfer factor, Eq. (21)
υ = kinematics viscosity

Superscripts and Subscripts

$*$ = dimensionless
e = equivalent
i = inlet
o = outlet
max = maximum
min = minimum

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