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Transient Response of FGM Pressure Vessels

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Abstract The present study aims to investigate the transient behavior of thick-walled cylinders under dynamic internal pressure. Analytical solutions are possible only for simple time-dependent pressure functions. The solution procedure presented is general in the sense that the pressure applied may be an arbitrary continuous function of time, impulsive or given in a discrete form. The material considered is isotropic and heterogeneous with properties varying in the radial direction termed as Functionally Graded Material (FGM). Laplace transform method is used and the inversion into the time domain is performed using the modified Durbin's method. Verification of the numerical procedure is performed by comparing the results with those of an analytical solution available in the literature for a simple exponentially-varying pressure. The inhomogeneity constant in the material property model is shown to have a significant effect on the transient response.

Keywords Functionally graded materials • Vibration • Pressure vessel

1 Introduction

Functionally graded materials (FGM) have seen increasing applications in engineering fields. The continuous spatial variation of physical properties in such materials has eliminated some adverse effects such as stress concentration and

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delamination encountered in ordinary composites. Exact vibration analyses of homogeneous cylinders using Laplace transform date back as early as 1940s (e.g. see Tranter [1]). A more recent detailed study on the subject is due to Ghosh [2] where axisymmetric vibration of thick cylinders under various continuous dynamic pressures is investigated.

As for the dynamic response of heterogeneous cylinders; the works by Loy et al. [3] and Pradhan et al. [4] include studies using Rayleigh-Ritz method on the vibration of cylindrical shells made of functionally graded materials composed of stainless steel and nickel or zirconia. The transient waves in a functionally graded cylinder are analyzed by Han et al. [5] using a hybrid numerical method. A finite-element vibration analysis of FGM thick hollow cylinders is done by Shakeri et al. [6] where the cylinder is assumed to be made of many subcylinders.

The present paper presents solutions for the axisymmetric forced vibrations of a functionally graded isotropic thick-walled cylinder. The solutions are obtained in the Laplace domain and inverse transformation to the time domain is done by the modified Durbin's method. For an internal dynamic pressure continuous with time analytical solutions may be obtained through the calculus of residues. This analysis is viable only for simple cases of pressure functions. A benchmark solution given by Keles [7] is used to verify the numerical procedure. In cases of impulsive and piecewise continuous pressures it becomes inevitable to resort to numerical inversion schemes. The method chosen in the present work, Durbin's numerical inverse Laplace transform method, has been efficiently implemented in vibration analysis of structural elements (e.g. see Temel [8]). If the pressure is given in a point-by-point manner its closed-form Laplace transform will not be available and, for such cases, Durbin's method is also a valid, efficient procedure.

Elastic modulus and the density of the cylinder are assumed to vary radially according to a power-law and, for the sake of simplicity, constant Poisson's ratio is assumed throughout. Dynamic response is presented for various values of the inhomogeneity parameter. The analysis presented is readily applicable to FGM disks and hollow spheres.

2 Governing Equations

Consider a thick-walled hollow cylinder (or a disk) of inner radius a and outer radius ka where k is a constant. The cylinder is isotropic and made of functionally graded material. Modulus of elasticity and the density vary through the thickness as $E(r) = E_0 r^\beta$ and $\rho(r) = \rho_0 r^\beta$, respectively, where the exponent β essentially represents the degree of inhomogeneity. Under axisymmetric conditions using the strain-displacement relations, constitutive equations for plane strain and equation of motion in the radial direction gives the governing differential equation for the radial displacement $\bar{v}$ in the Laplace domain as

$$\frac{d^2\bar{v}}{dx^2} + \frac{m_1}{x}\frac{d\bar{v}}{dx} + \left(\frac{m_2}{x^2} - p^2\right)\bar{v} = 0 \tag{1}$$

where p is the Laplace parameter. The boundary conditions are

$$\overline{\sigma}_r|_{x=1} = -\overline{P}(p) \qquad \overline{\sigma}_r|_{x=k} = 0 \tag{2}$$

The solution of Eq. 1 is given in terms of Bessel functions as

$$\bar{v}(x, p) = x^{\varphi}(C_1 I_n(px) + C_2 K_n(px)) \tag{3}$$

where I_n and K_n are Bessel functions of first and second kind, respectively, of order n with $\varphi = -\frac{\beta}{2}$, $n = \sqrt{1 - \frac{\mu\beta}{1-\mu} + \frac{\beta^2}{4}}$

The constant C_1 and C_2 are evaluated using the boundary conditions. Making use of the recurrence formulae [9] the complete solution in the Laplace domain is written as

$$\bar{v}(x, p) = -\frac{\bar{P}(p)}{C_{11}} x^{\varphi} \frac{F(p)}{G(p)} \tag{4}$$

where

$$\begin{aligned} F(p) = {} & [K_n(pk)S_1 + pk K_{n-1}(pk)] I_n(px) \\ & - [I_n(pk)S_1 - pk I_{n-1}(pk)] K_n(px) \end{aligned} \tag{5}$$

and

$$\begin{aligned} G(p) = {} & [K_n(pk)S_1 + pk K_{n-1}(pk)][I_n(p)S_1 - p I_{n-1}(p)] \\ & - [K_n(p)S_1 + p K_{n-1}(p)][I_n(pk)S_1 - pk I_{n-1}(pk)] \end{aligned} \tag{6}$$

where $S_1 = (n - m - \varphi)$ and $m = C_{12}/C_{11}$

To find the final solution in real time domain, the inverse Laplace transform needs to be performed on Eq. 4. For the internal pressure $P(\tau) = P_0(1-e^{\gamma\tau})$, a benchmark solution given by Keles [7] will be used for comparison purposes.

3 Solutions by the Modified Durbin's Method

For impulsive and piecewise continuous time-dependent pressures the calculus of residues method seizes to be a viable option; numerical methods are inevitably employed.

A numerical inverse Laplace transform technique to obtain the values in the time domain is Durbin's inverse Laplace transform technique based on the Fast Fourier Transform (FFT) algorithm. Durbin's formulation for inverse Laplace transform is summarized as follows [10]:

The function $f(t)$ at time t_j is given by

$$f(t_j) \cong \frac{2Exp[aj\Delta t]}{T}\left[-\frac{1}{2}\mathrm{Re}\left\{\bar{F}(a)\right\} + \mathrm{Re}\left\{\sum_{k=0}^{N-1}(\bar{F}(p_k)L_k)Exp\left[i\left(\frac{2\pi}{N}\right)jk\right]\right\}\right]$$
$$(j = 0, 1, 2, \ldots, N-1) \tag{7}$$

where $\bar{F}(p_k)$ is the Laplace transform of $f(t)$. The kth Laplace parameter is defined as $p_k = a + ik2\pi/T$. The number N is $N = T/\Delta t$ where T is the solution interval and Δt is the time increment which is taken as $\Delta T = 0.1$. The selection of constant a is done by assigning a value to αT. It is suggested that the value of αT be in the range 5–10. For the numerical examples presented in this paper this value is taken as 6. Finally, the results are modified by multiplying each term in the summation by Lanczos factor L_k as suggested in [11]

$$L_k = \sin\left(\frac{k\pi}{N}\right)\Big/\left(\frac{k\pi}{N}\right), \quad (L_0 = 1) \tag{8}$$

If the Laplace transform of the function $f(t)$ is not given in closed-form such as in the case of point-by-point definition, the discrete values need first to be transformed into the Laplace domain as follows:

$$\bar{F}(p_k) = \Delta t \sum_{n=0}^{N-1}\left[f(t_n)e^{-at_n}\right]e^{-i\frac{2\pi nk}{N}} \tag{9}$$

For various pressures only the term $\overline{P}(p)$ is changed in the solution given by Eq. 4. To verify the numerical procedure, the benchmark solution is resolved by numerical inversion and virtually an exact match is obtained within six-digit accuracy by using the first 20 natural frequencies inherent in the solution.

4 Results and Conclusions

Forced vibration analysis of FGM hollow cylinders has been performed using Laplace transform technique. The material is assumed to be graded with respect to a power-law whose exponent represents the degree of inhomogeneity. A constant Poisson's ratio of 0.3 has been used throughout. For a rectangular wave pressure

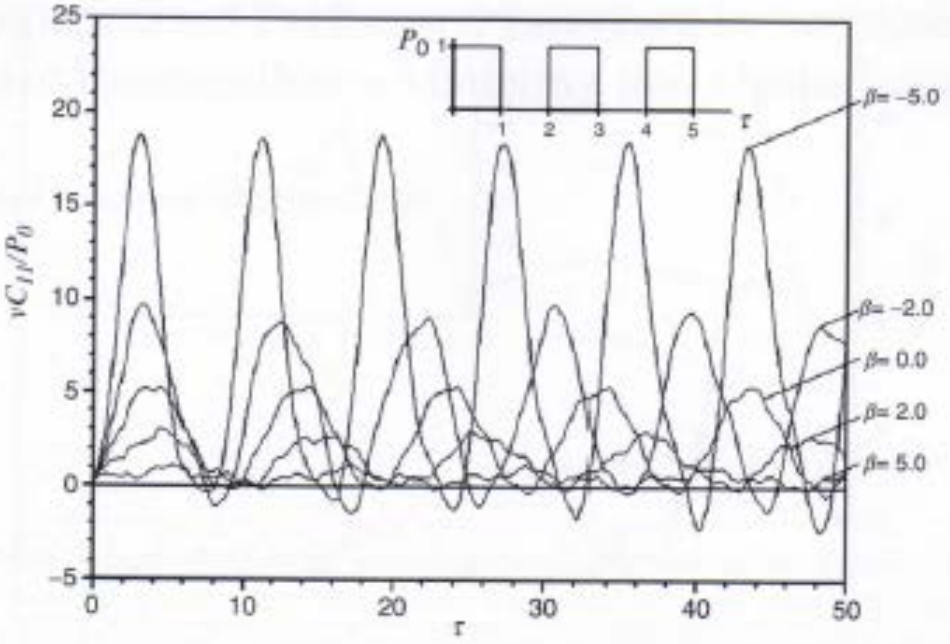


Fig. 1 Radial displacement on inside surface subjected to periodic rectangular pressure for $k = 2.0, \beta = -5.0, -2.0, 0.0, 2.0, 5.0$

$P(t) = 2P_0 \sum_{k=0}^{\infty} (-1)^k U(t - 2k)$, where $U(t - 2k)$ is the Heaviside's function, the Laplace transform is given in closed-form as $\overline{P}(p) = 2P_0/[p(1 + Exp(-2p))]$. This pressure is substituted in Eq. 4, and the numerical inversion into the real-time domain is performed by Durbin's method. The radial displacement on the inner surface is presented in Fig. 1 as a function of the non-dimensional time parameter. As mentioned in the preceding sections the pressure may be impulsive and given discretely. As an example of this case, consider the impulsive inner pressure which is present only up to $\tau = 12.0$ as shown in Fig. 2. After this point, there is no pressure and only free oscillations are present. Using Eq. 9 the real-time pressure values at time t_n read from the graph are transformed into the Laplace domain. Substituting these values into Eq. 7 yields the discrete solution in the time domain. The results are displayed in Fig. 2.

The values of the radial displacement decrease when β increases. A positive inhomogeneity constant refers to increasing stiffness in the radial direction which leads to the conclusion that it provides a stress shielding effect. It is obvious that a negative β would create a stress amplification effect.

The inhomogeneity constant, which includes continuously varying volume fraction of the constituents, is empirically determined and is a useful parameter from a design point of view in that it can be tailored for specific applications to control the stress distribution.

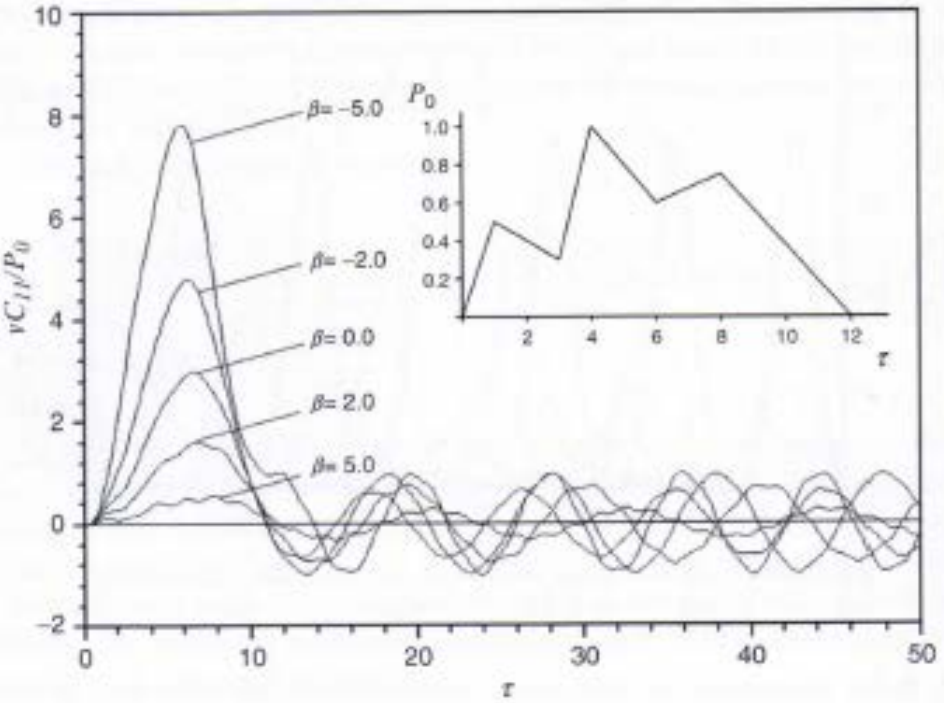


Fig. 2 Radial displacement on inside surface subjected to impulsive pressure given discretely. $k = 2.0, \beta = -5.0, -2.0, 0.0, 2.0, 5.0$

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