IMSEC 2017
2nd International Mediterranean Science and Engineering Congress (IMSEC 2017)
Çukurova University, Congress Center, October 25-27, 2017, Adana / TURKEY
Pages: 211-218, Paper ID:103

In-Plane Forced Vibration of Axially Functionally Graded Circular Arches

Ahmad Reshad Noori[1*], Timuçin Alp Aslan[2], Beytullah Temel[3]

[1]Cukurova University/Department of Civil Engineering, Turkey; a.reshadnoori@yahoo.com
[2]Cukurova University/Department of Civil Engineering, Turkey; taslan@cu.edu.tr
[3]Cukurova University/Department of Civil Engineering, Turkey; btemel@cu.edu.tr

Abstract

This paper presents a unified approach of Laplace transform and the Complementary Functions Method (CFM) for investigating the forced vibration behavior of axially functionally graded (AFG) arches with uniform cross-section subjected to in-plane time-depended loads. The material properties, Young's modulus and density of the arch vary continuously in the axial direction with respect to a power-law form and Poisson's ratio is assumed to be constant. A computer program is coded in Fortran to obtain the results of the proposed method. Current results are then compared with the results of ANSYS to establish the validity and reliability of the presented approach.

Keywords: Axially functionally graded arches; Laplace Transform; Complementary Functions Method (CFM); Forced Vibration.

1. INTRODUCTION

Axially functionally graded materials (AFGMs) are advanced materials which their material properties show a continuous variation along the axis of the structure. AFG Circular arches are used as structural elements in civil, mechanical, aerospace, marine and other many important engineering structures. Generally those structures are subjected to time-dependent loads during their service life. Thus, assessing the forced vibration of AFG circular arches is indispensable.

Elishakoff and Guede [1] discussed the vibration and buckling of simply supported axially graded beams. Aydogdu and Taskin [2] studied the free vibration of simply supported beams. Aydogdu [3] analyzed the vibration and buckling of axially functionally graded simply supported beams by using the semi-inverse method. A single 4^{th} order governing partial differential equation is derived by Li [4] for analyzing static and dynamic behaviors of FGM beams. Free vibration and the dynamic response of a FG simply-supported beam under a concentrated moving harmonic load are analyzed by Simsek and Kocatürk [5].

A new beam theory is used by Sina et al. [6] to analyze the free vibration of functionally graded beams. In-plane free vibration of functionally graded (FG) thin-to-moderately thick deep circular arches in thermal environments is presented by Malekzadeh et. al. [7] based on first-order shear deformation theory (FSDT). Filipich and Piovan [8] developed a model for the for dynamic analysis FGM thick curved beam according to the theory of strength of materials.

Huang and Li [9] studied the free vibration analysis of axially functionally graded beams with non-uniform cross-section by a novel and simple approach based on the integral equation method. Free vibration and stability analysis of axially functionally graded tapered Timoshenko beams are studied by Shahba et. al. [10] through a finite element approach.

Yousefi and Rastgoo [11] presented a formulation for the free vibration response of FGM spatial curved beam while the material properties are considered to be graded in the direction of the curvature of the curved beam. Şimşek et. al. [12] examined the linear dynamic response of an axially functionally graded (AFG) beam with simply supports subjected to a moving harmonic by using Euler–Bernoulli beam theory. Shahba and Rajasekaran [13] studied the free vibration and stability of axially functionally graded tapered Euler–Bernoulli beams. Piovan et. al. [14] introduced a general model for in-plane and out-of-plane dynamics and buckling analysis of functionally graded circular curved beams.

Wu et. al. [15] suggested an effective approach to examine the free in-plane vibration analysis of a curved beam (arch) with arbitrary various concentrated elements. The free and forced vibration analysis of FG nanobeams are examined by Uymaz [16] based on the nonlocal elasticity. Akgöz and Civalek [17] investigated the vibration analysis of FGM and non-uniform micro beams by Bernoulli–Euler beam and modified couple stress theory. Eroglu [18] studied the in-pla-

(*) Corresponding author

ne free vibrations of FGM circular beams in thermal environment. Static and forced vibration response of moderately thick deep functionally graded curved beams are investigated by Kurtaran [19] with the aid of generalized differential quadrature method.

Çalım [20] analyzed the free vibration and transient response of axially functionally graded (AFG) Timoshenko beams on two-parameter viscoelastic foundation by using the CFM, in the Laplace domain. The forced vibration behavior of axially functionally graded (AFG) Timoshenko beams of varying cross-section is examined by Çalım [21] through the unified approach of the CFM and Laplace transform. Noori et. al. [22] investigated the dynamic behavior of homogeneous, isotropic and elastic stepped circular arches under various in-plane time-dependent loads in the Laplace domain by the CFM. Temel et. al. [23] examined the transient response of planar arches with the CFM in the Laplace domain.

Although the free vibration of FGM beam and arches has been considered very extensively in the literature, according to the best of the authors' knowledge, the transient response of AFG curved arches has not yet been reported in the available works; the aim of this study is to fill this gap.

In the present study, forced vibration response of AFG circular arches is investigated in conjunction with Timoshenko beam theory and unified approach of the CFM and Laplace transform. The unified approach of CFM method and Laplace transform has utilized to obtain accurate results of the transient response of the AFG circular arches. The governing equations of motion of the considered structures have obtained in the time domain at first. Then the Laplace transform has implemented and the CFM has employed to the canonical form of the first order ordinary differential equations in the Laplace domain. For the numerical solution of differential equations, the fifth-order Runge–Kutta (RK5) method has been applied. To transform the solutions from the Laplace domain to the time domain an appropriate inverse numerical Laplace transform method has been used. To further verify the results of the presented procedure, structural displacements and structural forces components are calculated and compared with results of ANSYS.

2. MATERIAL AND METHOD

2.1 Functionally Graded Material

Consider an elastic AFG circular arch with a uniform circular cross-section. The Young's modulus, and mass density of the circular arch vary continuously only in the axial direction along the curvature of the arch *(ϕ)*, while the Poisson's ratio is assumed to be constant.

$$E = E(\phi) = E_0 e^{\beta\phi} \quad ; \quad \rho = \rho(\phi) = \rho_0 e^{\beta\phi} \tag{1}$$

Where E_0 and ρ_0 are the values of the Young's modulus, and mass density at the mid-point of the arch $(\phi = 0)$. β is a constant describing the variation of material properties along the axial direction of the arch, and (β = 0) corresponds to an isotropic homogeneous material.

2.2 Governing Equations

The time and location depended partial differential equations of circular arches subjected to dynamic loads arches given as follows:

$$\frac{\partial U_t}{\partial \phi} = U_n + r\frac{T_t}{E(\phi)A} \tag{2}$$

$$\frac{\partial U_n}{\partial \phi} = -U_t + r\grave{U}_b + r\frac{T_n \alpha_n}{G(\phi)A} \tag{3}$$

$$\frac{\partial \grave{U}_b}{\partial \phi} = r\frac{M_b}{E(\phi)I_b} \tag{4}$$

$$\frac{\partial T_t}{\partial \phi} = r\rho(\phi)A\frac{\partial^2 U_t}{\partial t^2} + T_n - p_t \tag{5}$$

$$\frac{\partial T_n}{\partial \phi} = r\rho(\phi)A\frac{\partial^2 U_n}{\partial t^2} - T_t - p_n \tag{6}$$

$$\frac{\partial M_b}{\partial \phi} = r\rho(\phi)I_b\frac{\partial^2 \grave{U}_b}{\partial t^2} - \grave{T}_n - m_b \tag{7}$$

$$G(\phi) = \frac{E(\phi)}{2(1+\nu)} \tag{8}$$

Here, $E(\phi)$, $\rho(\phi)$, h, A, I_b, α_n, r, p_t, p_n and m_b indicate the modulus of elasticity, mass density, radius of the cross-section, cross-section area, the moment of inertia, shear correction factor, radius of curvature, distributed axial load, distributed perpendicular load and distributed moment of bending respectively.

The unknown column matrix, $\{Y(\phi,t)\}$, for the transient response of AFG circular arches is given as:

$$\{Y(\phi,t)\} = \{U_t, U_n, \dot{U}_b, T_t, T_n, M_b\}^T \tag{9}$$

The Laplace transform of a time-dependent function $f(t)$ is the function "$\overline{F}(s)$".

$$L[f(t)] = \overline{F}(s) = \int_0^\infty f(t)e^{-st}dt \tag{10}$$

where "s" is the Laplace variable also known as operator variable in the Laplace domain. The Laplace transform of first and second derivatives with respect to time are as follows:

$$L\left[\dot{f}(t)\right] = s\overline{F}(s) - f(0) \tag{11}$$

$$L\left[\ddot{f}(t)\right] = s^2\overline{F}(s) - sf(0) - \dot{f}(0) \tag{12}$$

Applying the Laplace transform to equations (2-7), converts these partial differential equations to variable-coefficient ordinary differential equations. Thereby, the governing ordinary differential equations of the dynamic behavior of in-plane loaded AFG circular arches can be obtained in the Laplace domain as follows:

$$\frac{d\overline{U}_t}{d\varphi} = \overline{U}_n + r\frac{\overline{T}_t}{E(\varphi)A} \tag{13}$$

$$\frac{d\overline{U}_n}{d\varphi} = -\overline{U}_t + r\overline{\Omega}_b + \frac{\overline{T}_n\alpha_n}{G(\varphi)A} \tag{14}$$

$$\frac{d\overline{\Omega}_b}{d\varphi} = r\frac{\overline{M}_b}{E(\varphi)I_b} \tag{15}$$

$$\frac{d\overline{T}_t}{d\varphi} = rs^2\rho(\varphi)A\overline{U}_t + \overline{T}_n - r\overline{p}_t \tag{16}$$

$$\frac{d\overline{T}_n}{d\varphi} = rs^2\rho(\varphi)\overline{U}_n - \overline{T}_t - r\overline{p}_n \tag{17}$$

$$\frac{d\overline{M}_b}{d\varphi} = rs^2\rho(\varphi)I_b\overline{\Omega}_b - r\overline{T}_n - r\overline{m}_b \tag{18}$$

Where the terms shown by $\overline{(\bullet)}$ indicates the Laplace transform of the quantities.

$$\begin{aligned}
L\left[\rho(\phi)A\frac{\partial^2 U_t}{\partial t^2}\right] &= \rho(\phi)A\left[s^2\overline{U}_t - sU_t(\phi,0) - \frac{\partial U_t(\phi,0)}{\partial t}\right] \\
L\left[\rho(\phi)A\frac{\partial^2 U_n}{\partial t^2}\right] &= \rho(\phi)A\left[s^2\overline{U}_n - sU_n(\phi,0) - \frac{\partial U_n(\phi,0)}{\partial t}\right] \\
L\left[\rho(\phi)I_b\frac{\partial^2 \dot{U}_b}{\partial t^2}\right] &= \rho(\phi)I_b\left[s^2\overline{U}_b - s\dot{U}_b(\phi,0) - \frac{\partial \dot{U}_b(\phi,0)}{\partial t}\right]
\end{aligned} \tag{19}$$

In this study the intial conditions for $t = 0$ are considered to be zero, which are the second and third terms on the ri-

ght-hand side of the Eq. (19). Here ϕ is independent variable and s indicates the parameter of the Laplace transform. In the numerical solution of the initial-value problem based on the CFM the fifth-order Runge–Kutta (RK5) algorithm is used which is one of the most efficient numerical methods (see Temel et al. [23]).

The results, obtained in the Laplace domain, are transformed to the time domain with the help of modified Durbin's numerical inverse Laplace transform method (Durbin [24], Temel et al. [25]).

3. NUMERICAL EXAMPLES AND DISCUSSION

In this study, a computer program is prepaed by FORTRAN programming language to examine the undamped forced vibration of AFG circular arches of uniform cross-section area. The numerical results of the present study are compared with results obtained by ANSYS to demonstrate the accuracy and performance of the presented procedure.

A fixed-ended elastic AFG circular arch, shown in figure 1.a, is now assumed under a point time-dependent step load, given in figure 1.b with an amplitude of p0 = 1 kg, applied to its midpoint. Material and geometric properties of arch are assumed to be density, ρ0=7850x10-6 kgf/cm3, Poisson's ratio, ν = 0.3, and modulus of elasticity, E0=2.1x106 kgf/cm2. Shear correction factor αn = 1.11, radius of curvature of the circular arch r = 100 cm, radius of the cross section of the arch h = 1 cm and $\phi 0 = \pi/4$.

Flexural rigidity and cross section of the arch are;

$$E(\phi)I_b = E(\phi)\frac{\pi h^4}{4} \qquad A = \pi h^2$$

The boundary conditions of the symmetric point and the fixed-end are given as follows:

$$\phi = 0 \rightarrow \begin{cases} U_t = 0 \\ \dot{U}_b = 0 \\ T_n = -p/2 \end{cases} \qquad \phi = \phi_0 \rightarrow \begin{cases} U_t = 0 \\ U_n = 0 \\ \dot{U}_b = 0 \end{cases}$$

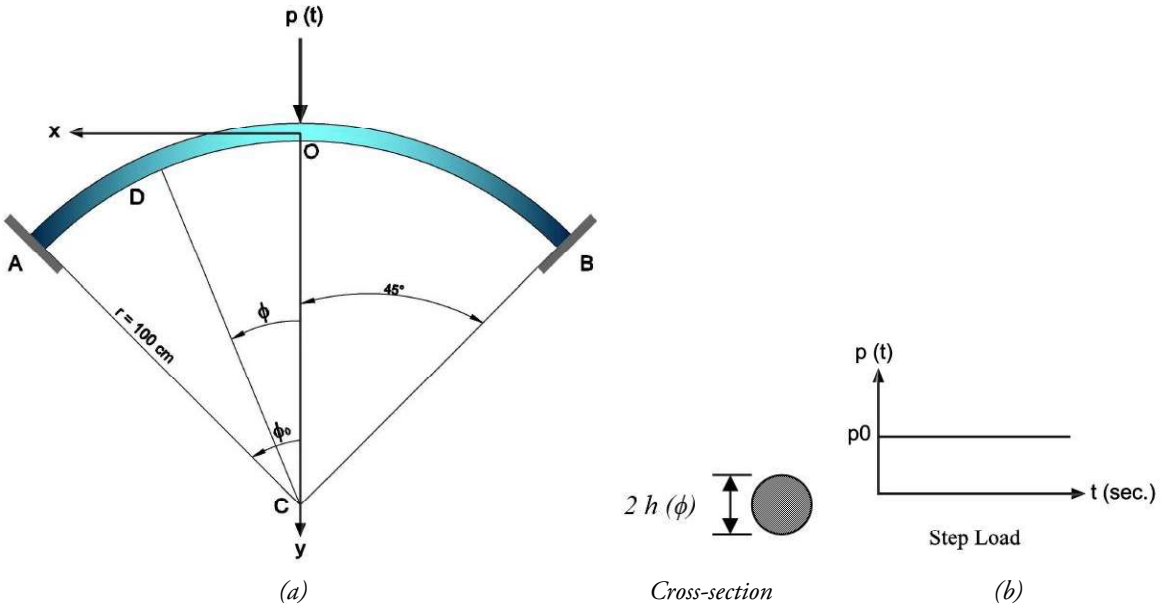


Figure 1: (a) Fix-ended AFG circular arch; (b) Time-dependent step load;

To analyze the AFG arch with the traditional axially layered model in ANSYS a parametric study has been done to determine the sufficient number of layers for having a continuous variation of the material properties. To obtain the forced vibration behavior, the AFG arch is divided into 30, 90 and 270 elements and comparisons are illustrated in figures 2 – 3. The material gradient index is assumed to be β =0.5.

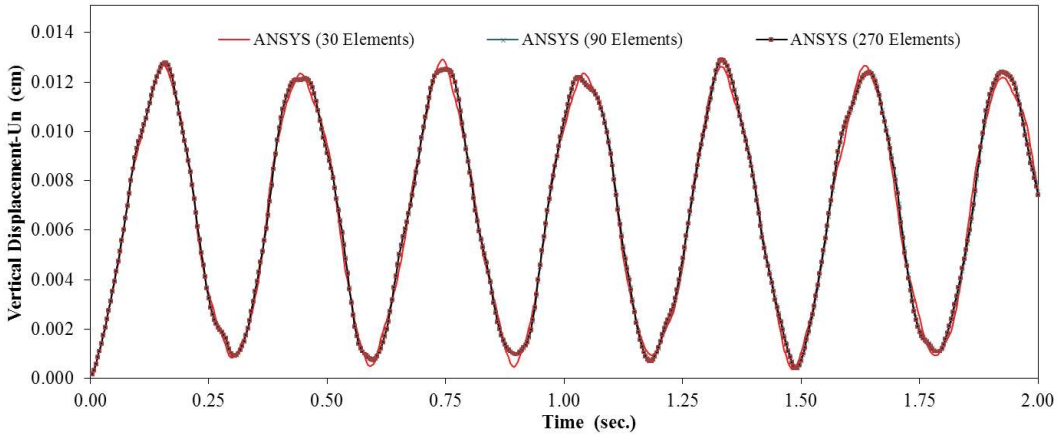


Figure 2: Vertical displacement of the midpoint of the AFG circular arch under a point step load.

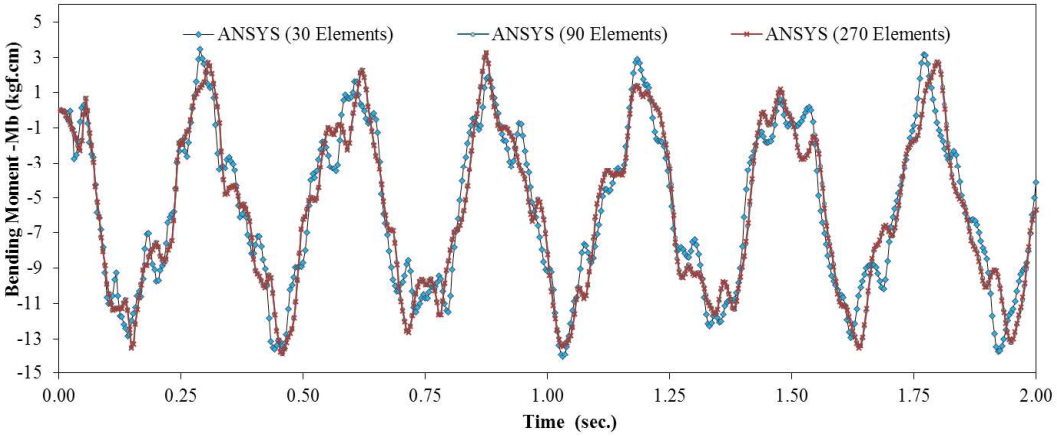


Figure 3: Moment of bending at the arch fixed-end under a point step load.

As can be seen in the figures 2 and 3 the arch must be divided into 270 or more finite elements to provide consistent result in ANSYS. For this reason, 270 finite elements will be used to mesh the AFG arch thereinafter in ANSYS.

Based on our previous researches Temel et. al. [23] it is apparent that in the forced vibration cases, results obtained for a coarse time increment along with fewer Laplace transform parameters overlap the results obtained with finer increments and higher parameters by the presented procedure which indicates the efficiency and exactness of the present method while the accuracy of the ANSYS results is directly proportional to the appropriate selection of the optimum time increment and number of sub steps. In this study 64 and 512 sub steps have been used in presented procedure and ANSYS respectively. Time-dependent behavior of structural displacements, forces and moments of the AFG arch are shown in figures 4 – 6. The material gradient index is considered to be β =0.5.

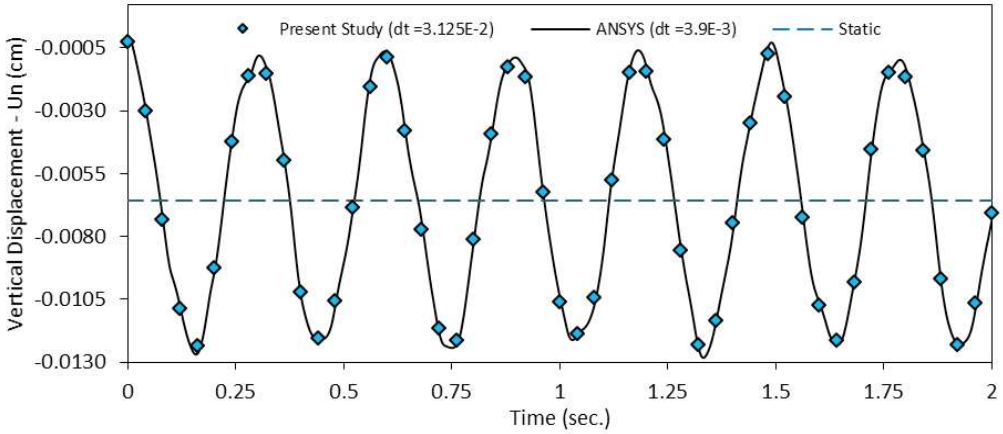


Figure 4: Vertical displacement of the midpoint of the AFG circular arch under a point step load.

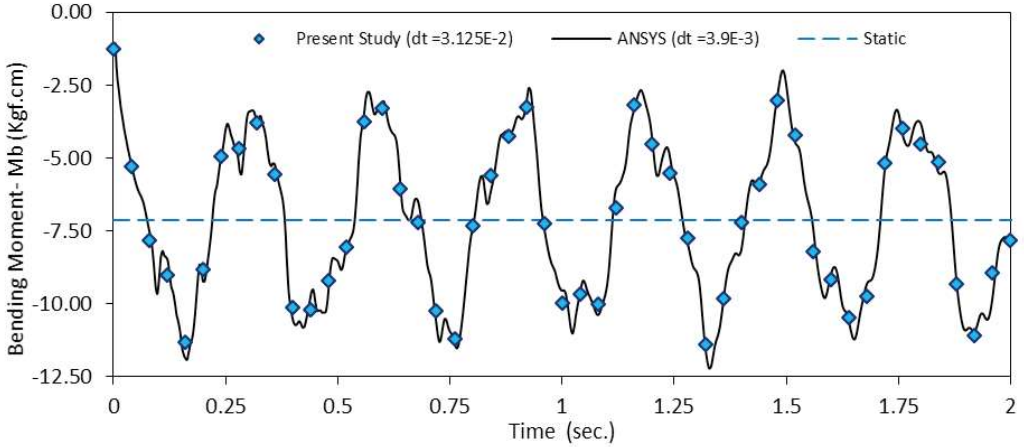


Figure 5: Moment of bending at the midpoint of the AFG circular arch.

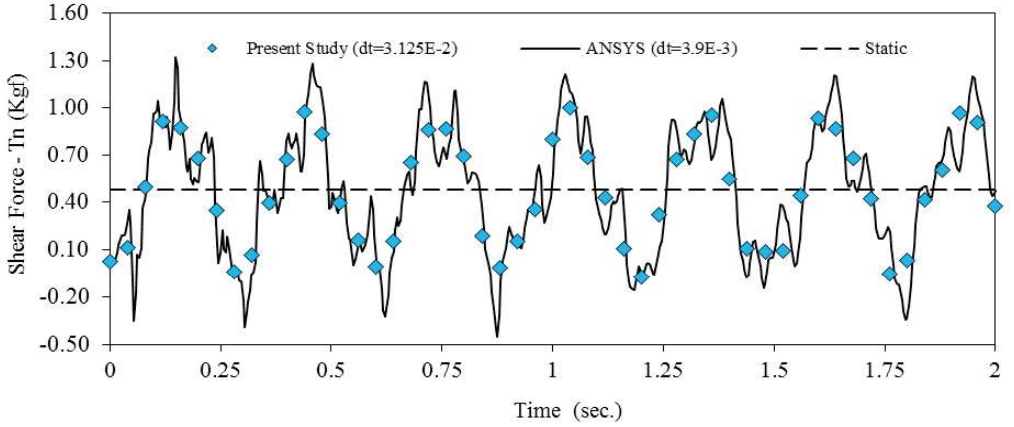


Figure 6: Shear force at the arch fixed-end under a point step load.

It is observed in figures 4 – 6 that the time-dependent deformation, structural forces and moment results obtained by the combination of Laplace transform and the CFM are far more efficient than the results of conventional step-by-step time integration method (ANSYS).

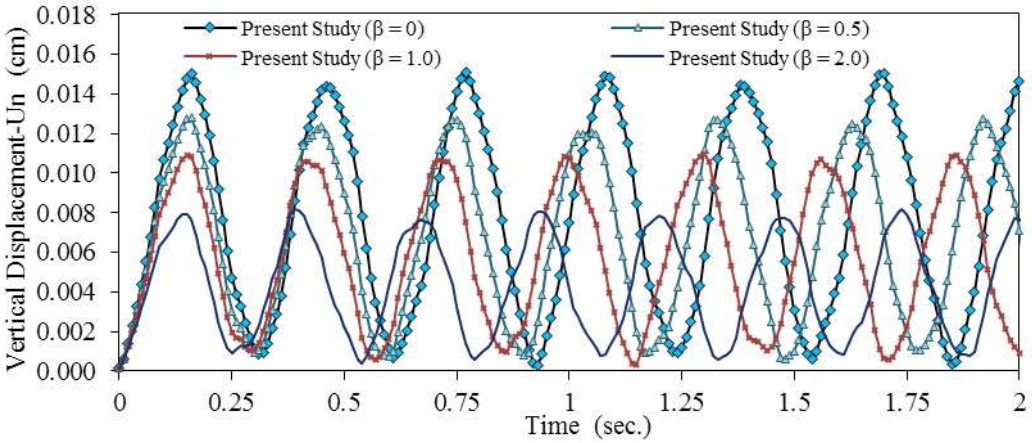


Figure 7: Vertical displacement of the midpoint of the AFG circular arch for various values of β.

After the verification of the proposed method, the transient response of the considered AFG circular arch is investigated for several types of materials which vary continuously in the axial direction with respect to a power-law form. Results are calculated and compared for various values of the material gradient index "β" to do a parametric study. Comparisons are presented in figures 7 – 9.

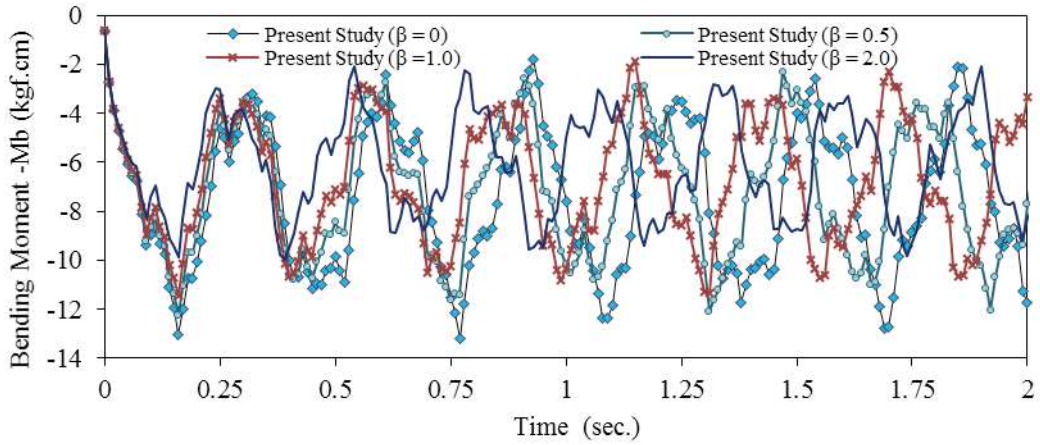


Figure 8: Moment of bending at the midpoint of the AFG circular arch for various values of β.

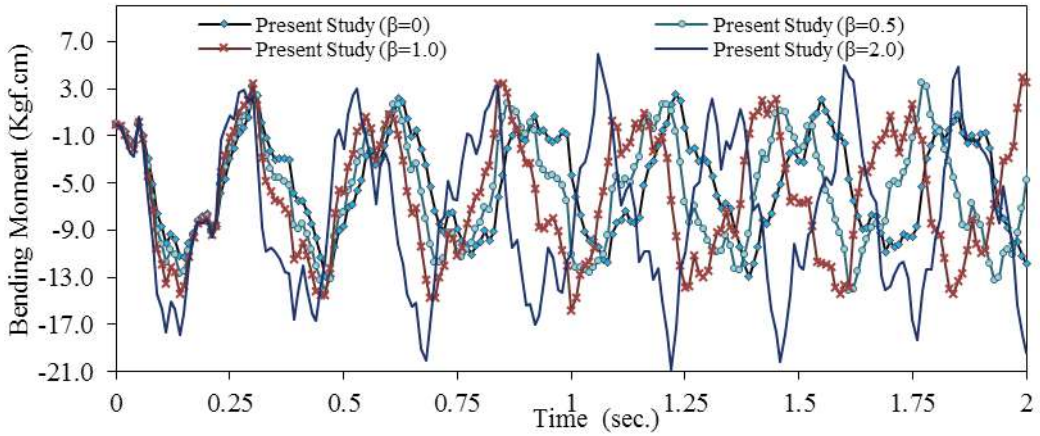


Figure 9: Moment of bending at the fixed-end of the AFG circular arch for various values of β.

It can be observed in figures 7 and 8 that displacement and bending moment of the midpoint of the arch are inversely proportional to the material gradient index, so that when the β is increasing the amplitudes of vertical displacement and bending moment of the arch midpoint are decreasing. In the fixed end of the arch there is the exact opposite situation as the material gradient index increases the magnitude of the bending moment increases as well, which can be clearly seen in figure 9.

4. CONCLUSION

The forced vibration analysis of AFG circular arches is carried out through the unified approach of Laplace transform and the CFM with required suitable integration step size as the effect of shear deformation is considered. Related initial value problems are solved numerical by RK5 algorithm. Governing equations are first obtained in the time domain and then transformed to the Laplace domain. The obtained set of simultaneous linear algebraic equations are solved by the CFM for a set of Laplace parameters and solutions are retransformed to the time domain by an efficient inverse numerical Laplace transform method. For the proposed procedure, a computer program is coded in FORTRAN. Results of the presented method are compared with those obtained from ANSYS. It is found that present method gives high accurate results even with fewer Laplace parameter and coarse time step size while a finer time increment is required to acquire true results by of conventional step by step time integration method.

It can be clearly understood from this research that the variation of the material gradient index has significant effect on the transient response of the AFG circular arches. Result of the presented paper could be compared and used in the future studies on the AFG in-plane loaded arches.

5. ACKNOWLEDGEMENTS

The authors thank the Scientific Research Projects Directorate of Cukurova University for supporting the present study (FDK-2017-8254) and the Scientific and Technical Research Council of Turkey (TÜBİTAK) for supporting the Ph.D. program of Ahmad Reshad Noori at Cukurova University.

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