ÇUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES

MSc THESIS

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MECHANICS OF COMPOSITE STRUCTURES USING MATLAB GUI

DEPARTMENT OF MECHANICAL ENGINEERING

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ABSTRACT

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The aim of this study is to design a user-friendly software package that carries out in-plane static analysis using Matlab GUI (Graphical User Interface). The program's potential users will be engineers and academicians who are especially engaged with fiber-reinforced composites. Some computer packages have been released to aid the analyses which are concerned with the design of composite structures. Since solving advanced problems of composite structures requires long processing time with the present computer programs such as C, Fortran, Pascal, Basic etc. and, also finite element packages remain rather cumbersome, software packages which are especially designed for composite materials are needed for efficient calculations. It is envisaged that the proposed thesis work will yield a computer program which addresses the technical needs in a highly visual, user-friendly manner. The designated software is able to analyse composite plates that are exposed not only to mechanical loads but also hygroscopic and thermal loads. In addition, it also provides the option to the user of performing failure analysis which is the conclusive step in designing a structural element.

Key Words: Composite Plate, Failure Criterion, Hygro-Thermal Loads, Matlab GUI, Plane Stress

ÖZ

YÜKSEK LİSANS TEZİ

Matlab GUI Kullanarak Kompozit Plakların Mekaniği

Sefa YILDIRIM

ÇUKUROVA ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ
MAKİNA MÜHENDİSLİĞİ ANABİLİM DALI

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Bu çalışmanın amacı, özellikle elyaf takviyeli kompozit malzemelerle ilgilenen Türk mühendis ve akademik camiasının rahatlıkla kullanabileceği, hem görsel hem de teknik açıdan tüm ihtiyaçlarını karşılayabilecek Matlab GUI (Graphical User Interface)' de hazırlanan kullanıcı dostu bir paket programın tasarlanmasıdır. Kompozitlerin uzay, hava, kara ve deniz uygulamalarında artan kullanımı, mühendisler için elyaf takviyeli kompozit yapıların tasarımında yeterli bilgi sahibi olma ihtiyacını ortaya çıkarmıştır. Bu ihtiyacı karşılamak ve mühendisler ile akademisyenlere kompozit yapıların tasarımında yol göstermek amacıyla şirketler tarafından birkaç ticari paket program satışa sunulmuştur. Hedeflenen tez çalışması bu paket programlardan daha kapsamlı, daha kullanıcı dostu ve çok daha ekonomik bir yazılım oluşturulmasıdır. Çünkü kompozit malzemelerin tasarımı ve analizi için kullanılan ileri düzey karmaşık problemlerin hızlı ve güvenilir çözümü için C, Fortran, Pascal, Basic gibi temel programlama dilleri kullanılması uzun işlem süreleri ve düzeltilmesi güç birçok hatayla karşılaşılması gibi istenmeyen durumları ortaya çıkarır. Ayrıca, sonlu elemanlar yöntemini kullanan paket programlar da bu amaç için çok zahmetli ve zaman alıcı kalmaktadır. Bu durum göz önüne alındığında kompozit malzemeler için özel olarak tasarlanmış paket programlara ihtiyaç duyulduğu görülmüştür. Tasarlanan programda mekanik etkilere maruz kalan kompozit plakların analizi gerçekleştirebileceği gibi, hem mekanik hem de hygrothermal (nem ve sıcaklık etkisi) etkilerin bir arada bulunduğu durumların incelenmesi mümkün olacaktır. İstendiği takdirde Tsai-Wu Tensör Kırılma Kriteri ile emniyet katsayıları da hesaplanabilecektir.

Anahtar Kelimeler: Düzlem Gerilme, Kompozit Plak, Kırılma Kriteri, Matlab GUI, Termal Yükler

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LIST OF ABBREVIATONS

σ : Stress
ε : Strain
v : Poisson's ratio
E : Elasticity modulus
$[Q]$: Stiffness matrix
$[S]$: Compliance matrix
$[T]$: Transformation matrix
$[R]$: Reuter matrix
α : Coefficient of thermal expansion
β : Coefficient of moisture absorbtion
C : Moisture absorption per unit weight of the lamina
u : Normal displacement
v : Shear displacement
w : Bending displacement
κ : Curvature
N : Force resultant
M : Moment resultant
$[A]$: In plane stiffness matrix
$[B]$: Coupling stiffness matrix
$[D]$: Bending stiffness matrix
$[\alpha]$: In plane compliance matrix
$[\beta]$: Coupling compliance matrix
$[\delta]$: Bending compliance matrix
V^f : Fiber volume fraction
X : $(\sigma_1)_{max}$ in tension
Y : $(\sigma_2)_{max}$ in tension
X' : $(\sigma_1)_{max}$ in compression
Y' : $(\sigma_2)_{max}$ in compression
S : $(\tau_{12})_{max}$ in shear
R : Reserve factor

1. INTRODUCTION

Materials are the basic elements of all natural and man-made structures. Technological progress is associated with continuous improvement of existing material properties as well as with the expansion of structural material classes and types. Usually, new materials emerge due to the necessity to improve structural efficiency and performance. In addition, new materials themselves as a rule, in turn provide new opportunities to develop updated structures and technology, while the latter challenges materials science with new problems and tasks (Vasiliev and Morozov, 2007). The traditional monolithic metals such as steel, aluminum etc. and their alloys may not always fulfill the needs of the advancing engineering practice of today. It can only be possible to practice the expected performance by combining several materials. For example, parts of satellites need to be dimensionally stable in space during temperature changes between -160 ℃ and$+93.3$ ℃. Monolithic materials cannot meet these requirements, thus, composites are brought about as the only material to satisfy them (Kaw, 2005).

1.1. Composite Materials

A composite is a structural material that is formed of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other. The constituents when combined, produce a material with characteristics different from the individual components. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix that are generally continuous. The reinforcing phase material may be in the form of fibers, particles, or flakes. Concrete reinforced with steel and epoxy reinforced with graphite fibers can be given as examples of composite systems.

The composites concept is not a human invention. Wood is a natural composite material consisting of one species of polymer — cellulose fibers with good strength and stiffness — in a resinous matrix of another polymer, the polysaccharide lignin. Nature makes a much better job of design and manufacture

than human do, although human was able to recognize that the way of overcoming two major disadvantages of natural wood — that of size (a tree has a limited transverse dimension), and that of anisotropy (properties are markedly different in the axial and radial directions) — was to make the composite material that we call plywood. Bone, teeth and mollusc shells are other natural composites, combining hard ceramic reinforcing phases in natural organic polymer matrices. Human was aware, even from the earliest times, of the concept that combining materials could be advantageous, pre-date the use of reinforced concrete by the Romans which foreshadowed the pre and post-tensioned reinforced concretes of our own era. However it is only in the last half century that the science and technology of composite materials have developed to provide the engineer with a novel class of materials and the necessary tools to enable him to use them advantageously (Harris, 1999).

Existing composite materials can be classified into two main groups.

The first group comprises composites that are known as filled materials. The main feature of these materials is the existence of some basic or matrix material whose properties are improved by filling it with some particles. Usually the matrix volume fraction is more than 50% in such materials, and material properties, being naturally modified by the fillers, are governed mainly by the matrix. As a rule, filled materials can be considered as homogeneous and isotropic. For example, traditional models of mechanics of materials developed for metals and other conventional materials can be used to describe their behavior. This group of composites is not focused on in this thesis.

The second group of composites which are of interest in the present thesis involves composites that are called reinforced materials. The basic components of these materials (sometimes referred to as advanced composites) are long and thin fibers possessing high strength and stiffness. The fibers are bound with a matrix material whose volume fraction in a composite is usually less than 50%. The main properties of advanced composites, due to which these materials find a wide application in engineering, are governed by fibers (Vasiliev and Morozov, 2007).

1.1.1. Fiber-reinforced Composites

Fiber-reinforced composites are traditionally used in especially the aerospace industries due to the high performance reinforcements of a thin diameter in a matrix material such as epoxy and aluminum. Graphite/Epoxy, Kevlar/Epoxy, and Boron/Aluminum are the some examples of fiber-reinforced composites.

As seen in Table 1.1, the fibers used in advanced composites have strength and stiffness far above those of traditional bulk materials. The high strengths of the glass fibers are due to processing that avoids the internal or surface flaws which normally weaken glass, and the strength and stiffness of the polymeric aramid fiber is a consequence of the nearly perfect alignment of the molecular chains with the fiber axis. Fiber reinforcement is very effective since many materials are much stronger and stiffer in fiber form then they are in a bulk form.

Table 1.1. Properties of Composite Reinforcing Fibers

Material	*E* (GPa)	σb (GPa)	εb (%	*ρ* (Mg/m3)	*E/ρ* (MN-m/kg)	σb / *ρ* (MN-m/kg)
E-glass	72.4	2.4	2.6	2.54	28.5	0.95
S-glass	85.5	4.5	2.0	2.49	34.3	1.8
Aramid	124	3.6	2.3	1.45	86	2.5
Boron	400	3.5	1.0	2.45	163	1.43
HS graphite	253	4.5	1.1	1.80	140	2.5
HM graphite	520	2.4	0.6	1.85	281	1.3

Here E is Young's modulus, σb is breaking stress, εb is breaking strain, and ρ is density.

These materials are not generally usable as fibers alone, and typically they are impregnated by a matrix material that acts to transfer loads to the fibers, and also to protect the fibers from abrasion and environmental attack. The matrix dilutes the properties to some degree, but even so very high specific (weight-adjusted) properties are available from these materials. Metal and glass are available as matrix materials, but these are currently very expensive and largely

restricted to research laboratories. Polymers are much more commonly used, with unsaturated styrene-hardened polyesters having the majority of low-to-medium performance applications and epoxy or more sophisticated thermosets having the higher end of the market.

Table 1.2 Some Mechanical Properties of a Unidirectional Lamina

Property	Symbol	Units	Glass/ Epoxy	Boron/ Epoxy	Graphite/ Epoxy
Fiber volume fraction	V_f		0.45	0.50	0.70
Longitudinal elastic modulus	E_1	GPa	38.6	204	181
Transverse elastic modulus	E_2	GPa	8.27	18.5	10.3
Major Poisson's ratio	ϑ_{12}		0.26	0.23	0.28
Shear modulus	G_{12}	GPa	4.14	5.59	7.17

1.2. Manufacturing of Composite Materials

Even though this thesis is not about manufacturing, it is necessary to know something of the nature of the main processing methods used in order to understand how what happens during processing may affect the properties of the product.

In recent years, particularly due to the interest generated in the automotive industry, there is more emphasis on the development of manufacturing methods that can support mass production rates. Compression molding, pultrusion, and filament winding represent three such manufacturing processes. Although they have existed for many years, investigations on their basic characteristics and process optimization started mostly in the middle of 1970s. Resin transfer molding (RTM) is another manufacturing process that has received significant attention in both aerospace and automotive industries for its ability to produce composite parts with complex shapes at relatively high production rates. With the introduction of automation, fast-curing resins, new fiber forms, high-resolution

quality control tools, and so on, the manufacturing technology for fiber-reinforced polymer composites has advanced at a remarkably rapid pace (Mallick, 2007).

1.2.1. Compression Molding

Compression molding is used for transforming sheet-molding compounds (SMC) into finished products in matched molds. The principal advantage of compression molding is its ability to produce parts of complex geometry in short periods of time. Nonuniform thickness, ribs, bosses, flanges, holes, and shoulders, for example, can be incorporated during the compression-molding process. Thus, it allows the possibility of eliminating a number of secondary finishing operations, such as drilling, forming, and welding. Compression molding is considered the primary method of manufacturing for many structural automotive components, including road wheels, bumpers, and leaf springs (Mallick, 2007).

1.2.2. Pultrusion Process

The pultrusion process represents a continuous transportation of fiber bandles through a resin matrix bath, following by a dropping of them into a preheated die or a set of dies. After curing process, where the part is changed from wet saturated reinforcement to a solid part the pultrusion is saw-cut to desired length. Pultrusion process results in parts with complex shapes, such as tubing, channels, I-beams, Z-sections and flat bars (Adrian and Gheorge, 2010).

1.2.3. Filament-winding Process

In a filament-winding process, a band of continuous resin-impregnated rovings or monofilaments is wrapped around a rotating mandrel and cured to produce axisymmetric hollow parts. Among the applications of filament winding are automotive drive shafts, helicopter blades, oxygen tanks, pipelines, spherical

pressure vessels, conical rocket motor cases, and large underground gasoline storage tanks (Mallick, 2007).

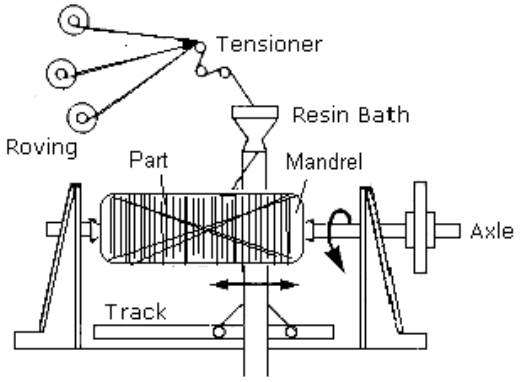


Figure 1.1. Schematic view of Filament-winding Process

1.2.4. Resin Transfer Molding Process

The RTM process is suitable for making small- to large-sized structures in small- to medium-volume quantities. RTM is used in automotive, aerospace, sporting goods, and consumer product applications. The structures typically made are helmets, doors, hockey sticks, bicycle frames, windmill blades, sports car bodies, automotive panels, and aircraft parts (Mazumdar, 2002).

In the RTM process, a preform is placed into the mold cavity. A matching mold half is mated to the first half and the two are clamped together. Then, using dispensing equipment, a pressurized mixture of thermoset resin, a catalyst, color, filler, etc., is pumped into the mold using single or multiple ports in the mold. After curing for 6 to 30 min, depending on the cure kinetics of the mixture, the part is then removed from the mold. Thus, RTM results in the production of structural parts with good surface finish on both sides of the part (Mazumdar, 2002).

2. PRELIMINARY WORK

Collier Research Corporation (1996) developed a software for composites called ST-Size at NASA which later evolved into HyperSizer that is design, analysis, and optimization software for composite and metallic structures.

Helius: CompositePro is put on the market by Firehole Technologies, Inc.(1996) which is a commercial windows program includes database, micromechanics, lamination theory, and beam, plate, and shell solutions.

Promal (Program for Micromechanical and Macromechanical Analysis of Laminates), the software tool which is specially designated to complement of the text book titled Mechanics of Composite Materials created by academician Autar K. Kaw (1997).

2.1. HyperSizer

HyperSizer is able to do discretely optimize in a manner, which guarantees structural integrity of the selected optimum design, using methods to accurately compute margins-of-safety for all potential failures. Optimization capabilities include finding minimum weight panel or beam concepts, material selections, cross sectional dimensions, thicknesses, and layups from a library of 50 different stiffened and sandwich designs and a database of composite, metallic, honeycomb, and foam materials. Any material or cross sectional dimension can be optimized to all load cases. Apply general edge loadings and/or boundary conditions through the Free Body Diagram software tab and solve for the resulting stresses and structural integrity using over 100 different failure analyses. Analyses include traditional industry methods and modern analytical and numerical solutions.

2.1.1. Panel and Beam Concepts

In HyperSizer different 50 panel and beam concepts are available. The engineer simply clicks which concepts to be considered. HyperSizer will concurrently optimize panel/beam design concepts, all cross section dimensions, material selections, and composite layups. The panels and beams are categorized into families.

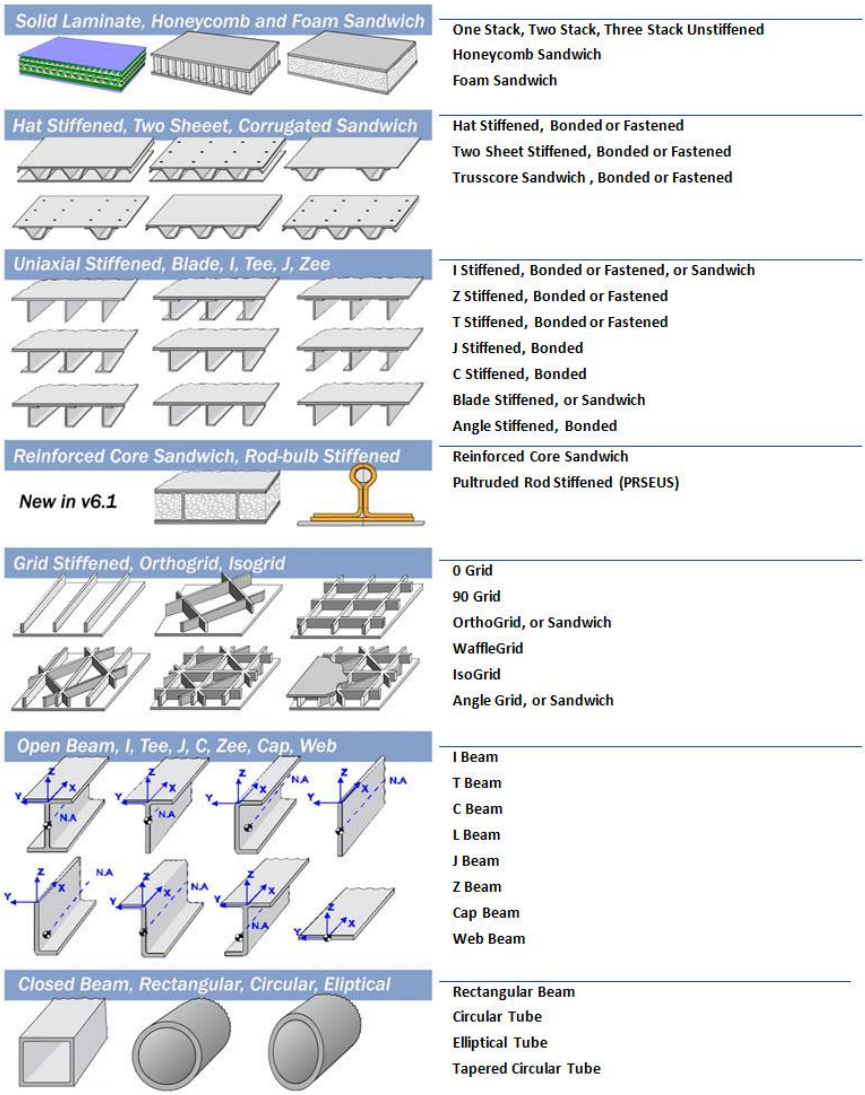


Figure 2.1. Sample Panel and Beam Concepts of HyperSizer

2.1.2. Design Optimization

In order to analyze an existing design or fabricated part, user can click on each design variable, and input values for the cross section dimension and sheet thickness. To optimize, the user enters a starting and an ending value for each variable. These starting and ending values are labeled in HyperSizer as minimum and maximum variable ranges. Next, the user determines how many permutations for each variable and then selects the different materials to attempt for the facesheet skin, sandwich core, etc. The factorial combination of each variable's permutation and material choices creates a large matrix of possible candidate designs.

The candidate designs are evaluated for strength and stability structural integrity for all applied load cases. If any strength or stability failure analysis indicates a negative margin-of-safety (a safety factor less than one) then that candidate design fails. The optimizer will quickly determine which candidate design is the lightest that passes all failure analyses.

2.1.3. Failure Analysis

Most analyses apply to all panel/beam concepts, such as buckling, cross section crippling, and modal frequency. However, design concepts have unique failure mechanisms. For a honeycomb panel, additional failures to consider are facesheet wrinkling and dimpling, and core shear and crushing. For stiffened panels, analyses are performed for each individual span segment, such as ply-by-ply material strength, local buckling, crippling, and flange to skin adhesive bonded joint strength. HyperSizer includes all of these types of failure analyses, and returns a margin-of-safety for each.

Documentation of analytical methods are presented in PDF format with the equations used for each failure analysis included along with verification models and validation test data. The user may click on a failure method and then

view the equations and the method described along with related verification data. General view of this module is indicated in Figure 2.2.

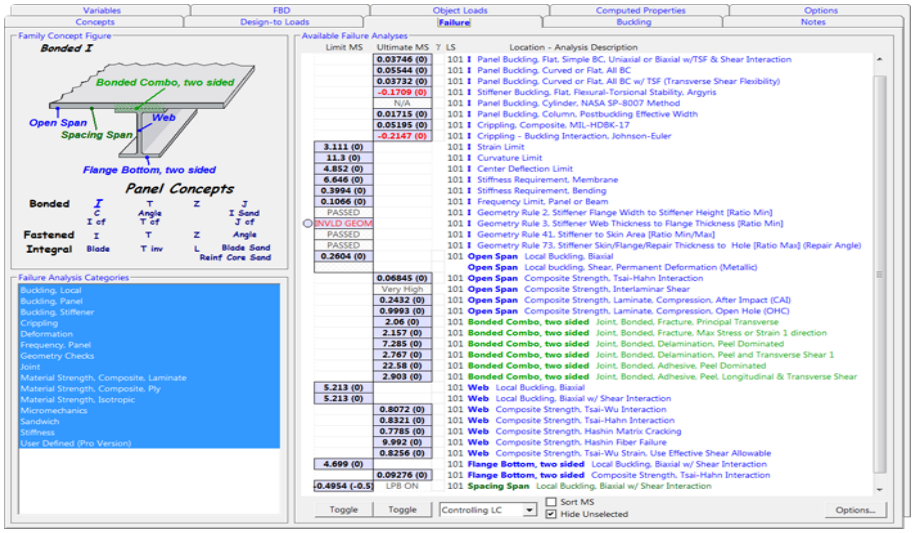


Figure 2.2. Failure Analysis Section of HyperSizer

2.2. Helius: CompositePro

Helius: CompositePro is not only one of those programs that simply allow user to calculate laminate properties and maybe stresses, strains, and failure criteria but also additionally to those basic functions, it includes micromechanics, structures (plates, sandwich laminates, beams, and shells), and various utilities (including some for filament winding and fabrics).

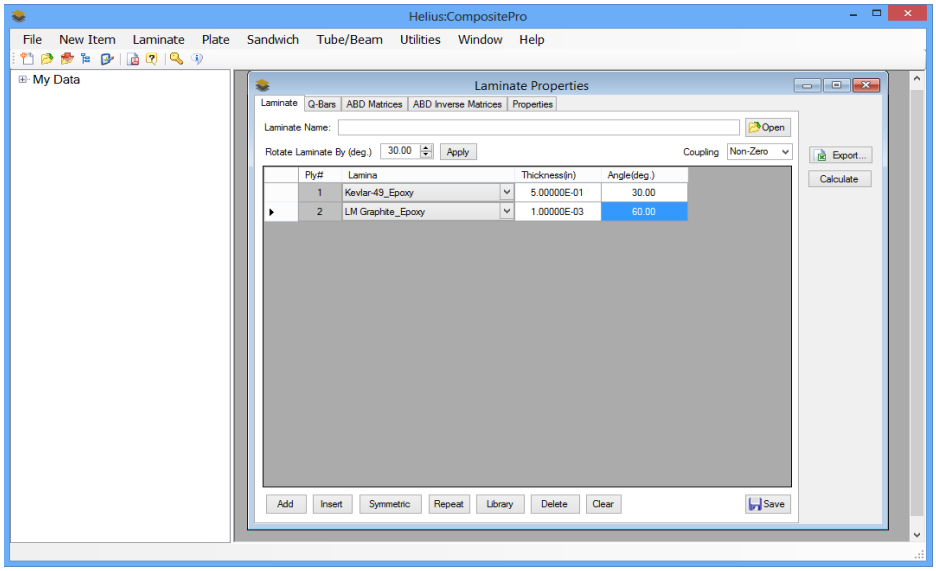


Figure 2.3. Main Interface of Helius: CompositePro

2.2.1. Materials Library

CompositePro has a material database of pre-characterized composite laminae that can be used to build laminates, in addition to numerous fiber, matrix, and core materials that can be used to create new composite materials, composite laminates and sandwich panels. Users can easily add new composite laminae, constituent materials and laminates to the material database.

CompositePro has also The New Item menu which provides options for creating new fiber, matrix and core materials as well as new lamina materials, new laminates or new thickness/angle templates. New materials defined by user are able to be saved to the Material Date File Tree for future use.

2.2.2. Laminate Analysis

The user can find nine different laminate analysis features from the laminate module to easily determine the properties of a laminate. A library of lamina properties can be used or user may input his own to create a layup of any combination.

Global laminate material properties can be easily calculated and plotted. Both 2D and 3D properties are generated. The laminate can then be loaded with in-plane strains, stresses, resultants, moment resultants and temperature or moisture loads. Stresses and strains on the individual laminae, due to the applied loads, can be displayed and plotted.

User may also select from 7 laminate failure criteria (max strain, max stress, Tsai-Wu, Tsai-Hill, Hashin, Christensen, Puck and MulitContinuum Theory (MCT)) to predict laminate strengths using the "First Ply" Failure, First Ply Failure Survey and Progressive Failure options.

2.2.3. Simple Structures

CompositePro provides three different modules for simple structures as plates, sandwich and tube/beam.

In plate module, three different options are available. Plate bending allows calculation of moments and deflections for simply supported uniform or concentrated loaded plates. The stability option calculates the minimum loads which will make a uniaxial or biaxial loaded plate buckle. Finally, the natural vibration frequency for a variety of boundary conditions can be determined.

In sandwich module, user may designate a laminate and then select core materials from a library or input own. The length, width and core thickness for the sandwich panel are then defined. Also there are three sandwich panel analysis features available in this module. Beam Bending determines the maximum transverse deflection due to a wide variety of transverse load types and boundary conditions. Panel Bending determines the transverse deflection of a simply supported sandwich plate to a uniformly distributed load. Stability lets user compute the critical buckling load of a plate subjected to uniaxial in-plane compression for a variety of boundary conditions.

In tube/beam module, user can test the behavior of composite tube or beam structures. Cross sections include solid plate, elliptical, circular, rectangular, I, C, U and T sections. Once the tube or beam is defined, there are five different

analysis options: Bending deflection, moments, stresses, etc. are calculated for point, uniform distributed and triangular loadings using combinations of fixed, free and simple supports. Torsional stresses and deflections can be determined. Fundamental vibration frequencies of bending and twisting are determined for a variety of end conditions and end masses. Cylindrical stability for external pressure, bending, torsion and axial compression is available as well as column stability calculations.

2.3. Promal

Promal is professionally developed software accompanying text book called Mechanics of Composite Materials by Autar K. Kaw with the aim of academic progression of students who taking courses of the mechanical behavior of composites. The software has 5 different programs which are shown in Figure 2.4 as Lamina Properties Database, Replacing Corrupted Lamina Properties Database, Macromechanics Analysis of a Single Lamina, Micromechanics Analysis of a Single Lamina, and Macromechanics Analysis of a Laminate(Kaw, 2012).

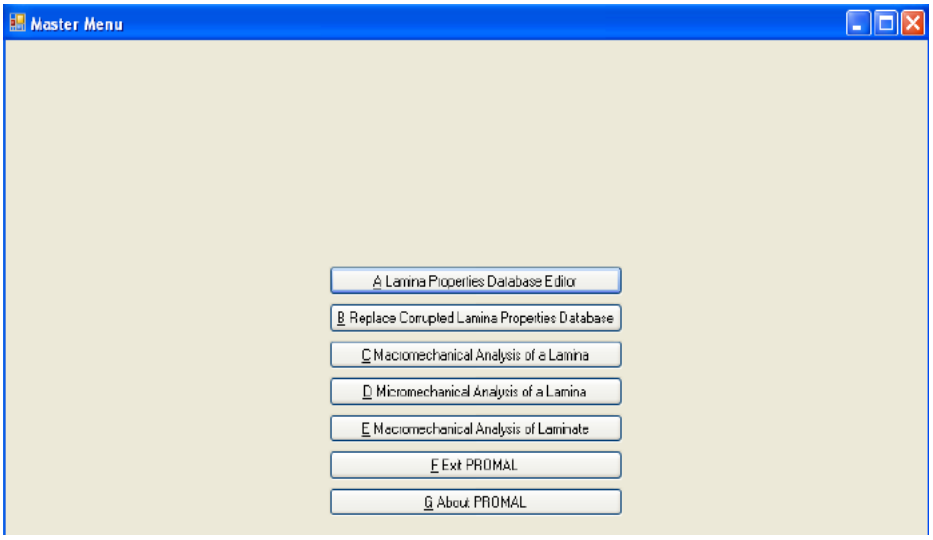


Figure 2.4. Master Menu Showing Available Programs in Promal

2.3.1. Lamina Properties Database

In this program, general view of which is shown below, the properties of unidirectional laminae can be added, deleted, updated, and saved. This is useful because these properties can then be loaded into other parts of the program without repeated inputs.

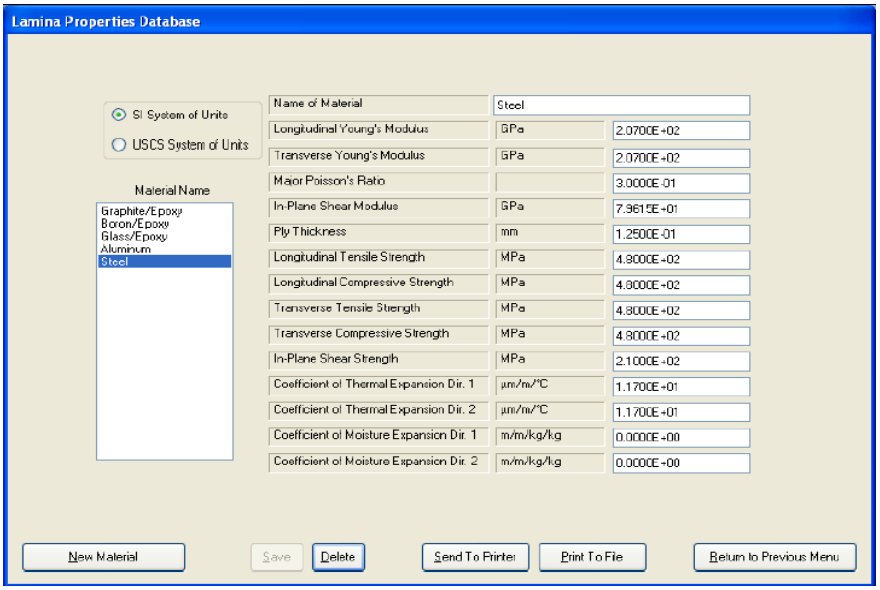


Figure 2.5. Menu for Updating and Adding Lamina Properties to Material Database

2.3.2. Replacing Corrupted Lamina Material Database

Sometimes, the material database for the lamina may get corrupted due to power shutdown, computer lockup, accidental rebooting or other reasons. This option allows user to replace the material database with the database that you received at the setup time.

2.3.3. Macromechanical Analysis of a Single Lamina

The second program conducts the macromechanical analysis of a lamina. The input mechanical properties of a lamina are chosen from the Lamina

Properties Database. User can then calculate the stiffness and the compliance matrices, the engineering constants and the strength ratios for an angle ply, and display the engineering constants and the strength ratios as a function of the ply angle in tabular or graphical form.

2.3.4. Micromechanical Analysis of a Single Lamina

The third program conducts the micromechanical analysis of a lamina using formulas from the strength of materials approach. User enter the elastic moduli, coefficients of thermal and moisture expansion of the fiber and the matrix from the keyboard. User can then calculate the elastic moduli, thermal and moisture expansion coefficients of a unidirectional lamina for a particular fiber volume fraction. The program also displays these properties in a tabular/graphical form illustrated below as a function of the fiber volume fraction.

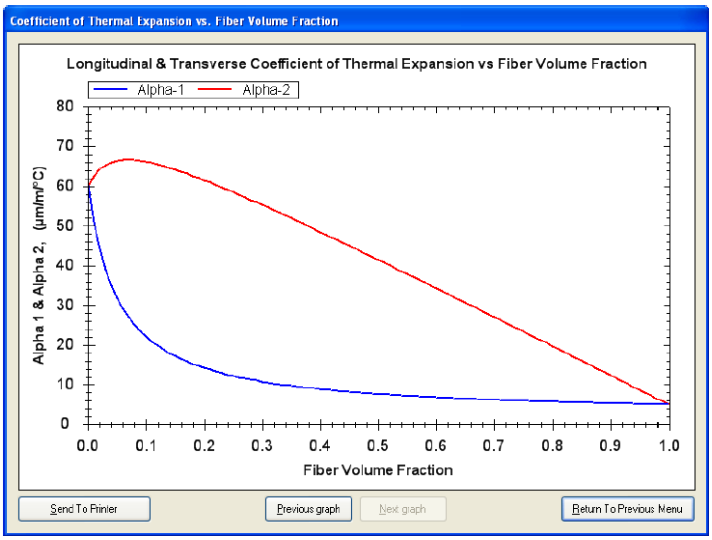


Figure 2.6. Coefficients of Thermal Expansion as a Function of Fiber Volume Fraction

2.3.5. Micromechanical Analysis of a Laminate

The fifth program shown below conducts the macromechanical analysis of a laminate. The laminate can be made of a single type of lamina, hybrid laminas,

and sandwich panels. The material properties for the lamina are selected from the material database of the second program. Next, user enter the orientation of the plies and the corresponding material number of each ply in the laminate. User can also read laminate stacking sequences from a data file. Now user can display the orientation and ply properties, the stiffness and the compliance matrices, and the equivalent plate properties of the laminate. Enter the mechanical and hygrothermal loads to calculate the global and local stresses and strains, strength ratios at the top, middle and bottom surface of each ply in the laminate.

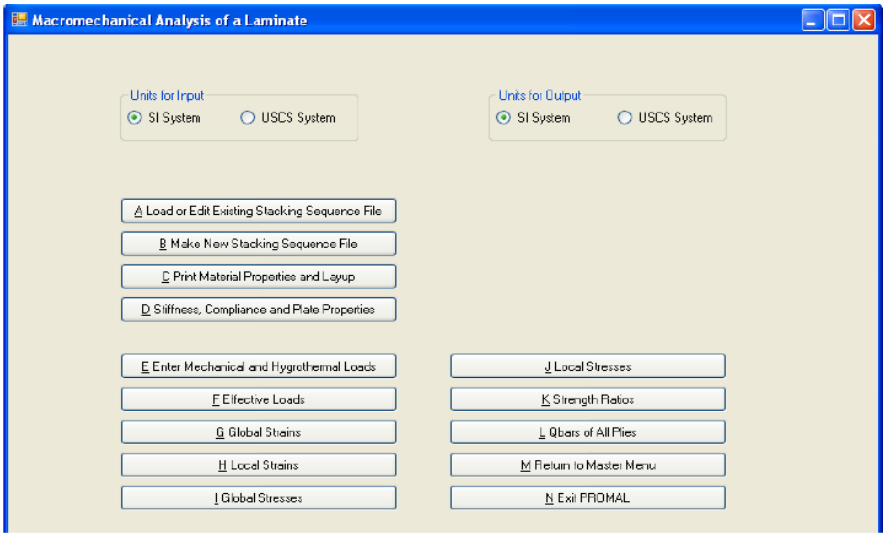


Figure 2.7. Main Menu for Macromechanical Analysis of a Laminate

3. MATERIAL and METHOD

In this section, the features of the software used and the theoretical background of the analysis will be presented.

3.1. Software Package Used in the Analysis: Matlab

The widely-used symbolic manipulator Matlab will be employed in the analysis and presentation of the results.

3.1.1. Matlab GUI

In this study, Matlab GUI is used to design of the software package called KomPro. A graphical user interface (GUI) is a pictorial interface to a program which provides the user with a familiar environment in which to work. This environment contains pushbuttons, toggle buttons, lists, menus, text boxes, and so forth, all of which are already familiar to the user, so that user can concentrate on using the application rather than on the mechanics involved in doing things. However, GUIs are harder for the programmer because a GUI-based program must be prepared for mouse clicks (or possibly keyboard input) for any GUI element at any time. Such inputs are known as events, and a program that responds to events is said to be event driven. The three principal elements required to create a Matlab Graphical User Interface are (Chapman, 2001);

i. *Components:* Each item on a Matlab GUI (pushbuttons, labels, edit boxes, etc.) is a graphical component. The types of components include graphical controls (pushbuttons, edit boxes, lists, sliders, etc.), static elements (frames and text strings), menus, and axes. Graphical controls and static elements are created by the function uicontrol, and menus are created by the functions uimenu and uicontextmenu. Axes, which are used to display graphical data, are created by the function axes.

ii. *Figures*: The components of a GUI must be arranged within a figure, which is a window on the computer screen. In the past, figures have

been created automatically whenever we have plotted data. However, empty figures can be created with the function figure and can be used to hold any combination of components.

iii. *Callbacks*: Finally, there must be some way to perform an action if a user clicks a mouse on a button or types information on a keyboard. A mouse click or a key press is an event, and the Matlab program must respond to each event if the program is to perform its function. For example, if a user clicks on a button, that event must cause the Matlab code that implements the function of the button to be executed. The code executed in response to an event is known as a callback. There must be a callback to implement the function of each graphical component on the GUI. The basic GUI elements are summarized in Table 3.1, and sample elements are shown in Figure 3.1.

Table 3.1. Matlab GUI Components

Element	Created By	Description
Graphical Controls		
Pushbutton	uicontrol	A graphical component that implements a pushbutton. It triggers a callback when clicked with a mouse.
Toggle button	uicontrol	A graphical component that implements a toggle button. A toggle button is either “on” or “off,” and it changes state each time that it is clicked. Each mouse button click also triggers a callback.
Radio button	uicontrol	A radio button is a type of toggle button that appears as a small circle with a dot in the middle when it is “on.” Groups of radio buttons are used to implement mutually exclusive choices. Each mouse click on a radio button triggers a callback.

Check box	uicontrol	A check box is a type of toggle button that appears as a small square with a check mark in it when it is “on.” Each mouse click on a check box triggers a callback.
Edit box	uicontrol	An edit box displays a text string and allows the user to modify the information displayed. A callback is triggered when the user presses the Enter key.
List box	uicontrol	A list box is a graphical control that displays a series of text strings. A user can select one of the text strings by single- or double-clicking on it. A callback is triggered when the user selects a string.
Popup menus	uicontrol	A popupmenu is a graphical control that displays a series of text strings in response to a mouse click. When the popupmenu is not clicked on, only the currently selected string is visible.
Slider	uicontrol	A slider is a graphical control to adjust a value in a smooth, continuous fashion by dragging the control with a mouse. Each slider change triggers a callback.
Static Elements		
Frame	uicontrol	Creates a frame, which is a rectangular box within a figure. Frames are used to group sets of controls together. Frames never trigger callbacks.
Text field	uicontrol	Creates a label, which is a text string located at a point on the figure. Text fields never trigger callbacks.
Menus and Axes		
Menu items	uimenu	Creates a menu item. Menu items trigger a callback when a mouse button is released over them.

Context menus	uicontextmenu	Creates a context menu, which is a menu that appears over a graphical object when a user right-clicks the mouse on that object.
Axes	axes	Creates a new set of axes to display data on. Axes never trigger callbacks.

And design module of Matlab GUI with some basic GUI elements are illustrated in Figure 3.1.

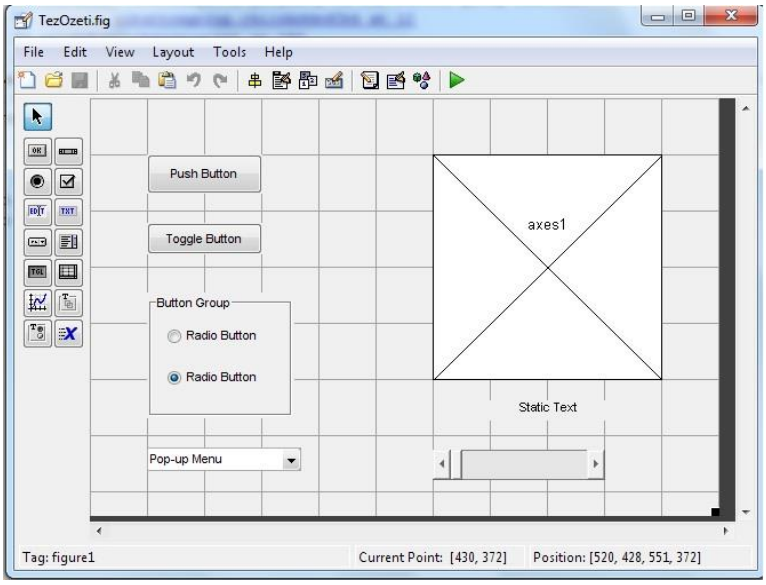


Figure 3.1. Sample Examples of Matlab GUI Elements

3.2. Theory

The Classical Laminated Plate Theory (CLPT) will be employed in the analysis of composite laminated plates.

3.2.1. Stress-Strain Relations for Plates

In the most general case, in an elastic material at a point the stress-strain relationship is;

$$\sigma_i = C_{ij}\varepsilon_i \qquad i,j = 1,2,\ldots,6 \tag{3.1}$$

Global coordinate system is selected in order to define the axis of plates which is illustrated in Figure 3.2

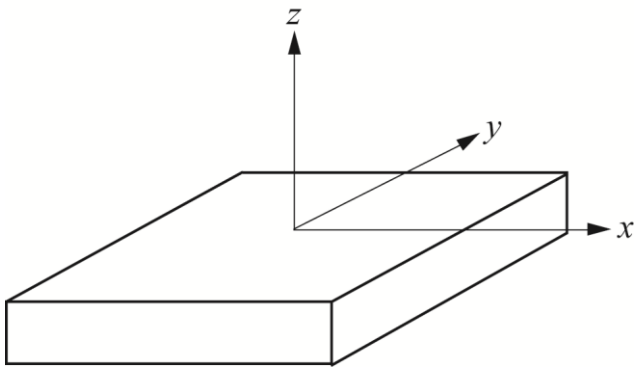


Figure 3.2. Coordinate Axes of a Plate

Equation (3.1) in matrix form;

$$\{\sigma\} = [C]\{\varepsilon\} \tag{3.2}$$

The stiffness matrix $[C]$ is symmetric. Hence, the general anisotropic material has 21 independent elastic moduli;

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \quad \text{(Triclinic material)} \tag{3.3}$$

If there is one plane of symmetry (3=0 plane), 13 independent elastic moduli;

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{21} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{31} & C_{32} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & C_{46} \\ 0 & 0 & 0 & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \quad \text{(Monoclinic material)} \tag{3.4}$$

If there are three mutually orthogonal planes of material symmetry (ex. unidirectional composites shown in Figure 3.3), 9 independent moduli;

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad \text{(Orthotropic material)} \qquad (3.5)$$

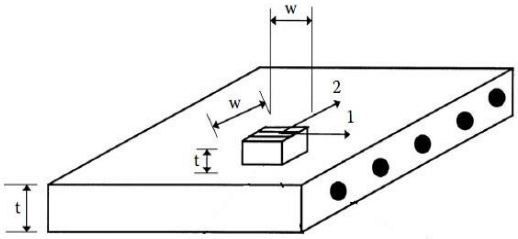


Figure 3.3. An On-Axis Unidirectional Lamina (orthotropic material)

3.2.2. Stress, Strain, Plain Stress for Specially Orthotropic Plates

If the applied loads are either parallel or perpendicular to the fibers then the plate is considered specially orthotropic that is shown in Figure 3.4.

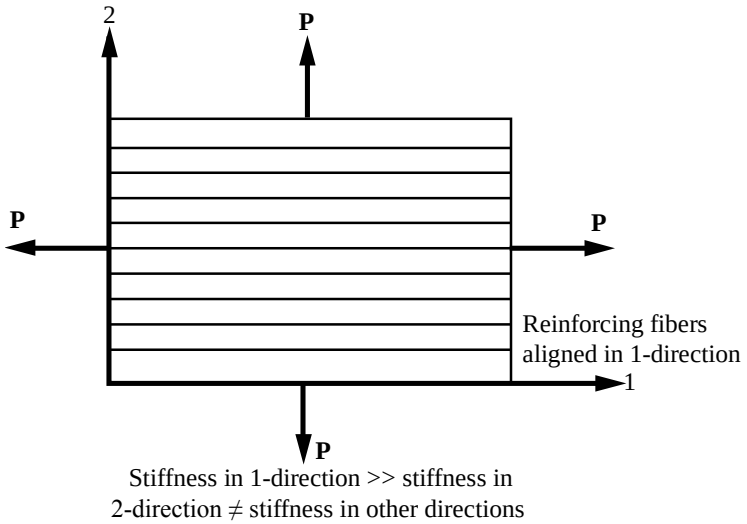


Figure 3.4. Orthotropic Fiber-Reinforced Lamina

In general, plates will be subjected to stresses in more than one direction within the plane. This is referred to as plane stress state. Poisson's ratio is defined

as the ratio of the strain perpendicular to a given loading direction to the strain parallel to this given loading direction;

Major Poisson's Ratio (for $\sigma_1 \neq 0$ only)

$$v_{12} = -\frac{\varepsilon_2}{\varepsilon_1} \tag{3.6}$$

Minor Poisson's Ratio (for $\sigma_2 \neq 0$ only)

$$v_{21} = -\frac{\varepsilon_1}{\varepsilon_2} \tag{3.7}$$

If we apply σ_1 only;

$$\varepsilon_1 = \frac{\sigma_1}{E_1}, \qquad \varepsilon_2 = -\, v_{12}\frac{\sigma_1}{E_1} \tag{3.8}$$

If we apply σ_2 only;

$$\varepsilon_2 = \frac{\sigma_2}{E_2}, \qquad \varepsilon_1 = -\, v_{21}\frac{\sigma_2}{E_2} \tag{3.9}$$

When σ_1 and σ_2 are both present;

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \, v_{21}\frac{\sigma_2}{E_2} \, , and \;\; \varepsilon_2 = \frac{\sigma_2}{E_2} - \, v_{12}\frac{\sigma_1}{E_1} \tag{3.10}$$

Shear forces can also be present. As normal stress and strains, shear stress and strains are related by a constant that is called the shear modulus and is usually denoted by G.

Thus;

$$\tau_{12} = \, \gamma_{12} G_{12} \tag{3.11}$$

Since it is known that a relationship exists between Poisson's ratios and the modulii in each of the two axes directions, namely;

$$v_{12}\, E_2 = \, v_{21}\, E_1 \tag{3.12}$$

Equations (3.11) and (3.12) can be written in matrix form as;

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{3.13}$$

Where;

$$S_{11} = \frac{1}{E_1} \ , \ S_{22} = \frac{1}{E_2}$$
$$S_{12} = S_{21} = -\frac{v_{21}}{E_2} \ , S_{66} = \frac{1}{G_{12}} \quad \text{(3.14a-d)}$$

Inverting we get;

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad \text{(3.15)}$$

Where;

$$Q_{11} = \frac{E_1}{1 - v_{12}v_{21}} \ , \ Q_{22} = \frac{E_2}{1 - v_{12}v_{21}}$$
$$Q_{12} = Q_{21} = \frac{v_{12}E_2}{1 - v_{12}v_{21}} \ , \ Q_{66} = G_{12} \quad \text{(3.16a-d)}$$

3.2.3. Stress, Strain, Plain Stress for Generally Orthotropic Plates

Generally, owing to their low stiffness and strength properties in the transverse direction, a laminate does not consist only of unidirectional laminae. Therefore, in most laminates, some laminae are placed at an angle. If the unidirectional lamina in Figure 3.4 is loaded at angle other than 0° or 90°, the lamina is now referred to as generally orthotropic in which, in general, the loading direction does not coincide with the principal material directions. It is thus necessary to develop the stress–strain relationship for an angle lamina (off-axis). The stresses and strains must now be transformed into coordinates that do coincide with the principal material directions. This can be accomplished using the free-body diagram in Figure 3.5. From free body diagram (a), and summing forces in the 1-direction;

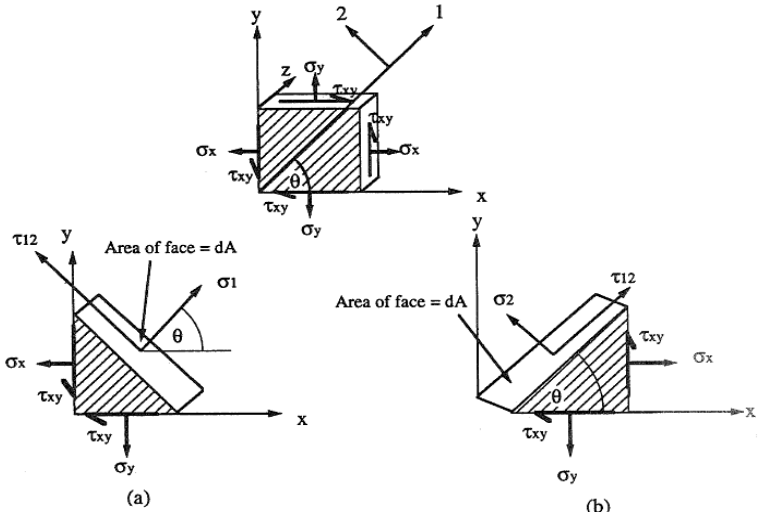


Figure 3.5. Generally Orthotropic Lamina

The axes in the 1–2 coordinate system are called the local axes (sometimes called the on-axis) and the axes in the x–y coordinate system are called the global axes (sometimes called the off-axis). The angle between the two axes is denoted by an angle θ.

The global and local stresses in an angle lamina are related to each other through the angle of the lamina (θ);

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{3.17}$$

Where, $[T]$ is called transformation matrix defined in matrix form as;

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & (m^2 - n^2) \end{bmatrix} \tag{3.18}$$

$$[T]^{-1} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & (m^2 - n^2) \end{bmatrix} \tag{3.19}$$

Here;

$$m = cos\theta, n = sin\theta \tag{3.20}$$

3.2.3.1. Transformation of Material Properties

Using the stress–strain Equation (3.15) in the local axes, Equation (3.17) can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1}[Q] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{3.21}$$

The local strains can be formed through the transformation matrix by means of the global strains;

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{Bmatrix} \tag{3.22}$$

which can be rewritten as;

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = [R][T][R]^{-1} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3.23}$$

Here $[R]$ donates Reuter Matrix that is defined as;

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{3.24}$$

Next, substituting Equation (3.23) into Equation (3.21) gives;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1}[Q][R][T][R]^{-1} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{3.25}$$

The first five matrices on the right side of the Equation (3.25) now gives the transformed stiffness matrix.

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \qquad ([Q]_{xy}=[\bar{Q}]_{12}) \tag{3.26}$$

The elements of the transformed reduced stiffness matrix $\bar{Q}$ and are given by;

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12}+2Q_{66})m^2n^2 + Q_{22}n^4\\
\bar{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})m^2n^2 + Q_{12}(m^4+n^4)\\
\bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12}+2Q_{66})m^2n^2 + Q_{22}m^4\\
\bar{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})m^3n + (Q_{12}-Q_{22}+2Q_{66})mn^3\\
\bar{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})mn^3 + (Q_{12}-Q_{22}+2Q_{66})m^3n\\
\bar{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})m^2n^2 + Q_{66}(m^4+n^4)
\end{aligned} \tag{3.27a-f}$$

As is seen from Equations (3.26) and (3.27a-f), a shear strain will cause normal stresses, and normal strains will contribute to a shear stress. This is referred to as extension-shear coupling if a lamina is loaded at an angle to the fibers other than 0° or 90°. That is, there will be coupling when the $\bar{Q}_{16}$ and/or $\bar{Q}_{26}$ terms in the lamina stiffness matrix are nonzero (Nettles, 1994).

For easier calculation;

$$\begin{aligned}
\bar{Q}_{11} &= U_1 + U_2 cos2\theta + U_3 cos4\theta\\
\bar{Q}_{22} &= U_1 - U_2 cos2\theta + U_3 cos4\theta\\
\bar{Q}_{12} &= U_4 - U_3 cos4\theta\\
\bar{Q}_{16} &= \frac{U_2}{2} sin2\theta + U_3 sin4\theta\\
\bar{Q}_{26} &= \frac{U_2}{2} sin2\theta - U_3 sin4\theta\\
\bar{Q}_{66} &= \frac{1}{2}(U_1 - U_4) - U_3 cos4\theta
\end{aligned} \tag{3.28a-f}$$

Where the invariants U are;

$$U_1 = \frac{1}{8}(3\bar{Q}_{11} + 3\bar{Q}_{22} + 2\bar{Q}_{12} + 4\bar{Q}_{66})$$
$$U_2 = \frac{1}{2}(\bar{Q}_{11} - \bar{Q}_{22})$$
$$U_3 = \frac{1}{8}(\bar{Q}_{11} + \bar{Q}_{22} - 2\bar{Q}_{12} - 4\bar{Q}_{66}) \quad (3.29a\text{-}d)$$
$$U_4 = \frac{1}{8}(\bar{Q}_{11} + \bar{Q}_{22} + 6\bar{Q}_{12} - 4\bar{Q}_{66})$$

Compliance terms can transform similarly;

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (3.30)$$

For easier calculation;

$$\bar{S}_{11} = V_1 + V_2 cos2\theta + V_3 cos4\theta$$
$$\bar{S}_{12} = V_4 - V_3 cos4\theta$$
$$\bar{S}_{22} = V_1 - V_2 cos2\theta + V_3 cos4\theta \quad (3.31a\text{-}f)$$
$$\bar{S}_{16} = V_2 sin2\theta + 2V_3 sin4\theta$$
$$\bar{S}_{26} = V_2 sin2\theta - 2V_3 sin4\theta$$
$$\bar{S}_{66} = 2(V_1 - V_4) - 4V_3 cos4$$

Where;

$$V_1 = \frac{1}{8}(3S_{11} + 3S_{22} + 2S_{12} + S_{66})$$
$$V_2 = \frac{1}{2}(S_{11} - S_{22})$$
$$V_3 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} - S_{66}) \quad (3.32a\text{-}d)$$
$$V_4 = \frac{1}{8}(S_{11} + S_{22} + 6S_{12} - S_{66})$$

3.2.4. Hygrothermal Stresses and Strains in a Lamina

Generally, composite materials are processed at high temperatures and then cooled down to room temperatures. For polymeric matrix composites, this temperature difference is in the range of 2000 ℃ to 300 ℃ and it may be an increase of 1000℃ for ceramic matrix composites. Since the coefficients of thermal expansion of the fiber and matrix are different, residual stresses result in a lamina when it is cooled down. Also, the cooling down induces strains due to the expansion in the lamina. In addition, most polymeric matrix composites can absorb or desorb moisture. This moisture change leads to swelling strains and stresses similar to those due to thermal expansion. Due to mismatch of the elastic constants and the coefficients of thermal expansion of the fiber and matrix, the hygrothermal strains are not equal in a lamina in the longitudinal and transverse directions (Kaw, 2005).

3.2.4.1. Hygrothermal Stress–Strain Relationships for a Unidirectional Lamina

For a unidirectional lamina, the stress–strain relationship with temperature and moisture difference gives

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}=\begin{bmatrix}S_{11} & S_{12} & 0\\ S_{21} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}+\begin{Bmatrix}{\varepsilon_1}^T\\ {\varepsilon_2}^T\\ 0\end{Bmatrix}+\begin{Bmatrix}{\varepsilon_1}^H\\ {\varepsilon_2}^H\\ 0\end{Bmatrix} \tag{3.33}$$

Where the superscripts T and H are used to denote temperature and moisture, respectively. Note that the temperature and moisture changes do not have any shearing strain terms because in the material axes no shearing strains are induced.

The thermally induced strains are given by

$$\begin{Bmatrix}{\varepsilon_1}^T\\ {\varepsilon_2}^T\\ 0\end{Bmatrix}=\begin{Bmatrix}\alpha_1\\ \alpha_2\\ 0\end{Bmatrix}\Delta T \tag{3.34}$$

Where α_1 and α_2 are the longitudinal and transverse coefficients of thermal expansion, respectively, and ΔT is the temperature change.

Thermal expansion coefficients transform as strains;

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \gamma_{xy}/2 \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \tag{3.35}$$

The moisture induced strains are given by

$$\begin{Bmatrix} {\varepsilon_1}^H \\ {\varepsilon_2}^H \\ 0 \end{Bmatrix} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix} C \tag{3.36}$$

Where $\beta_1$and $\beta_2$are the longitudinal and transverse coefficients of moisture, respectively, and C is the moisture absorption per unit weight of the lamina.

Coefficients of hygroscopic expansion transform as strains similar to thermal expansion coefficients.

$$\begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy}/2 \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix} \tag{3.37}$$

Total strain in general is;

$$\{\varepsilon\} = \{\varepsilon\}^M + \{\varepsilon\}^T + \{\varepsilon\}^H \tag{3.38}$$

Here the superscripts M, T and H are used to denote mechanical, thermal and hygroscopic, respectively.

3.2.5. Mechanics of Laminated Composites

The following assumptions are made;

i. The laminate thickness is very small compared to other dimensions
ii. The laminate layer are perfectly bounded

iii. Lines perpendicular to the surface of the lamina remain straight and perpendicular to the surface after the deformations

iv. The laminate and laminae are linear elastic

v. The through-the-thickness stresses and strains are negligible (plain stress state)



Figure 3.6. Laminated Structure of Composite Pipe

An example of a laminated structural element is shown in Figure 3.6.

A displacement of the plate in the x-direction is defined as u, which in the y-direction as v and for the z-direction w. Figure 3.7 shows these displacements.

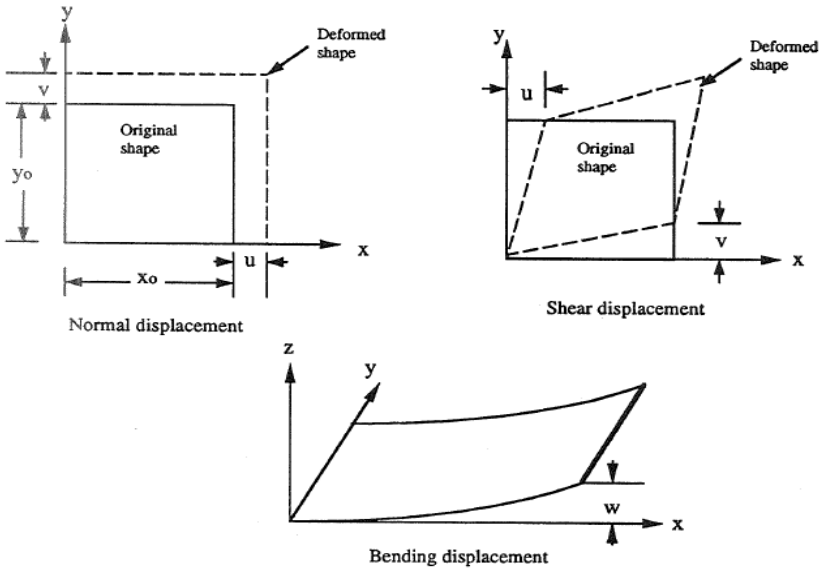


Figure 3.7 Displacements of a Plate

The strains are (small deformation gradients, infinitesimal strains);

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial u}{\partial y}, \quad \gamma_{xy} = (\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}) \tag{3.39}$$

The slopes of the plate if it is subjected to bending are;

$$\frac{\partial w}{\partial x} \quad along - x\ direction$$
$$\frac{\partial w}{\partial y} \quad along - y\ direction \tag{3.40a-b}$$

When examined at one edge of plate, total displacement is;

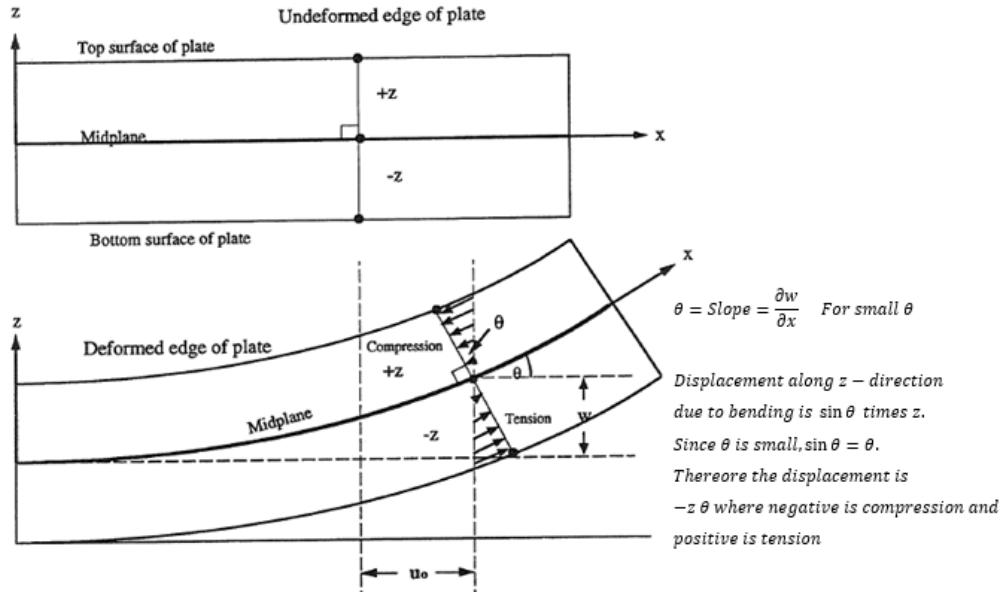


Figure 3.8 Total Displacement in a Plate

And we have in plane displacement u_0 due to in plane forces;

$$u = u_0 + \bar{u} \quad for\ z < 0$$
$$u = u_0 - \bar{u} \quad for\ z > 0 \tag{3.41a-b}$$

Total displacement in a plate and coordinates of plies are shown in Figure 3.8 and 3.9.

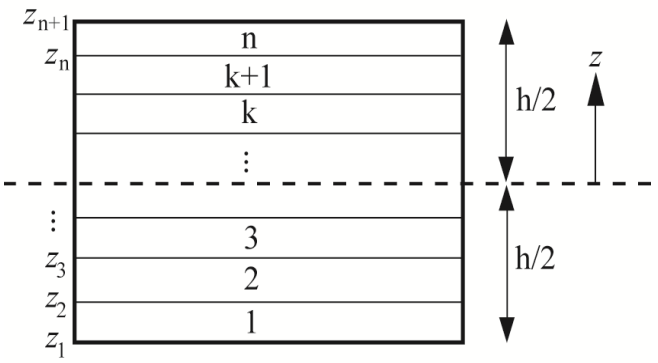


Figure 3.9 Coordinate locations of plies in a laminate

Thus;

$$u = u_0 - z\frac{\partial w}{\partial x} \tag{3.42}$$

Similarly;

$$v = v_0 - z\frac{\partial w}{\partial y} \tag{3.43}$$

Strains are then;

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial u}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} \tag{3.44a-c}$$

$$\gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right) = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2\,z\frac{\partial^2 w}{\partial x \partial y}$$

In matrix form;

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{3.45}$$

Here $\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}$ are the midplane strains and $\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$ are the plate curvatures.

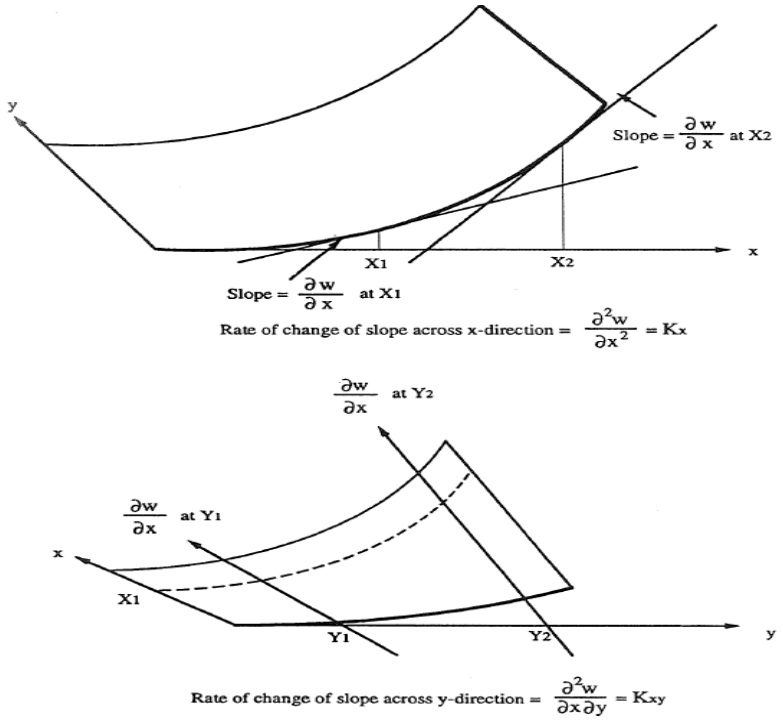


Figure 3.10 Definitions of Plate Curvatures

Finally, the stresses in each ply is given by;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (3.46)$$

3.2.5.1.Plate Equations

Force and moment resultants ($\{N\}\,(force/width)$, $\{M\}\,(moment/width)$) related to midplane strains and curvatures are illustrated in Figure 3.11.

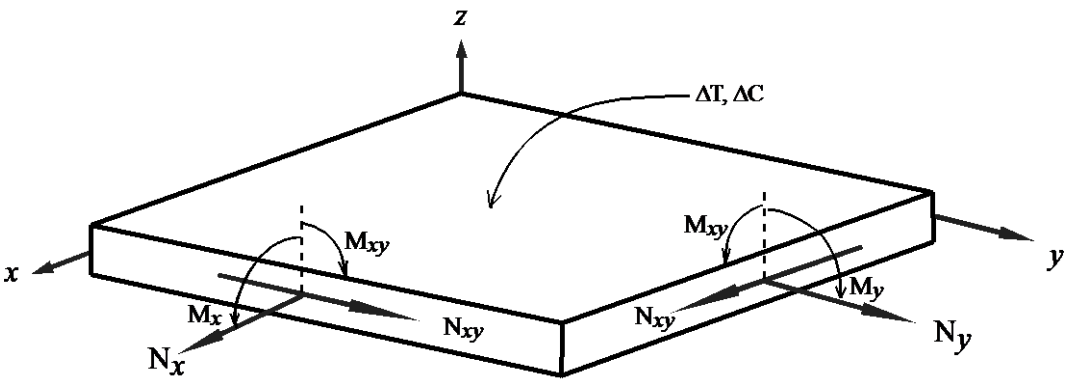


Figure 3.11. Force and Moment Resultants

Consider stress at a point through the thickness (Figure 3.12);

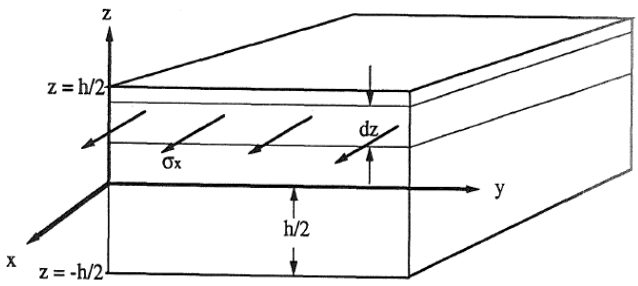


Figure 3.12. The Stress Acting on an Edge

Take σ_x as an example (Figure 3.13);

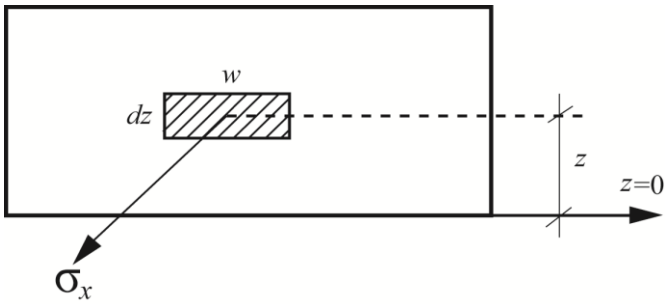


Figure 3.13. σ_x at a Point

No change along the w is assumed. Integrating the global stresses in each lamina gives the resultant forces per unit length in the x–y plane through the laminate thickness as;

$$N_x = \frac{1}{w}\int_{-h/2}^{h/2} \sigma_x dz w \;=>\; N_x = \int_{-h/2}^{h/2} \sigma_x dz \tag{3.47}$$

Similarly;

$$N_y = \int_{-h/2}^{h/2} \sigma_y dz$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} dz \tag{3.48a-b}$$

$\{N\}$ are called Force Resultants or Stress Flows.

Similarly, the moments are;

$$M_x = \frac{1}{w}\int_{-h/2}^{h/2} z\sigma_x dz w \;=>\; M_x = \int_{-h/2}^{h/2} \sigma_x z dz$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y z dz \tag{3.49a-c}$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z dz$$

Recall Equation (3.46) in a symbolic form;

$\{\sigma\} = [\bar{Q}]\{\varepsilon^0\} + z[\bar{Q}]\{\kappa\}$

Then;

$$\{N\} = \int_{-h/2}^{h/2} \left\{[\bar{Q}]\{\varepsilon^0\} + z[\bar{Q}]\{\kappa\}\right\} dz \tag{3.50}$$

Here;

$$\{N\} = \underbrace{\left(\int_{-h/2}^{h/2} [\bar{Q}]dz\right)}_{[A]}\{\varepsilon^0\} + \underbrace{\left(\int_{-h/2}^{h/2} [\bar{Q}]zdz\right)}_{[B]}\{\kappa\} \tag{3.51}$$

Similarly for moments;

$$\{M\} = \underbrace{\left(\int_{-h/2}^{h/2} [\bar{Q}]zdz\right)}_{[B]}\{\varepsilon^0\} + \underbrace{\left(\int_{-h/2}^{h/2} [\bar{Q}]z^2dz\right)}_{[D]}\{\kappa\} \tag{3.52}$$

In a matrix form;

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{3.53}$$

The $[A]$,$[B]$ and $[D]$ matrices are called the in plane, coupling, and bending stiffness matrices, respectively.

Where;

$$[A] = \sum_{k=1}^{N} [\bar{Q}]^k (z_{k+1} - z_k)$$

$$[B] = \frac{1}{2}\sum_{k=1}^{N} [\bar{Q}]^k ({z_{k+1}}^2 - {z_k}^2) \tag{3.54a-c}$$

$$[D] = \frac{1}{3}\sum_{k=1}^{N} [\bar{Q}]^k ({z_{k+1}}^3 - {z_k}^3)$$

If laminate stiffness matrix is inverted, compliance matrix will be obtained;

$$\begin{Bmatrix} \varepsilon^0 \\ \eta \end{Bmatrix} = \begin{bmatrix} [\alpha] & [\beta] \\ [\beta^T] & [\delta] \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \tag{3.55}$$

Here $[\beta]$ may not be symmetric however $\begin{bmatrix}[\alpha] & [\beta]\\ [\beta^T] & [\delta]\end{bmatrix}$ must be symmetric. According as stiffness matrix, the$[\alpha]$, $[\beta]$, and $[\delta]$ in plane, coupling, and bending compliance matrices as well. Note that $[B]$ is equal to zero for symmetric laminates and $[\beta]$ also.

3.2.5.2. Ply-by-ply Laminate Stress Analysis

Strain $\{\varepsilon\}$ is a continuous, linear function through the thickness along the laminate, shown in Equation (3.45) as

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix}$$

Stress $\{\sigma\}$ is continuous, linear through the lamina (a single layer) or a layer group with identical material and angle. However stress is not, in general, continuous through the laminate thickness.

The following are the steps for analyzing a laminated composite subjected to the applied forces and moments:

i. Calculate laminate stiffness matrix;

$$\begin{bmatrix}[A] & [B]\\ [B] & [D]\end{bmatrix} \tag{3.56}$$

ii. Invert this matrix to get compliance matrix;

$$\begin{bmatrix}[\alpha] & [\beta]\\ [\beta^T] & [\delta]\end{bmatrix} = \begin{bmatrix}[A] & [B]\\ [B] & [D]\end{bmatrix}^{-1} \tag{3.57}$$

iii. Calculate strains and curvatures (Equation (3.55));

$$\begin{Bmatrix}\varepsilon^0\\ \eta\end{Bmatrix} = \begin{bmatrix}[\alpha] & [\beta]\\ [\beta^T] & [\delta]\end{bmatrix}\begin{Bmatrix}N\\ M\end{Bmatrix}$$

iv. Calculate the local strains at a desired point z (Equation (3.45));

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix}$$

v. At the desired point z, transform off axis $\{\varepsilon_{xy}\}$ strains to on axis $\{\varepsilon_{12}\}$ strains (Equation (3.45));;

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{Bmatrix}$$

vi. Calculate on axis local stresses at the point of interest z;

$$\{\sigma\}_{12} = [Q]\{\varepsilon\}_{12} \quad (3.58)$$

(If off axis is desired, $\{\sigma\}_{xy} = [\bar{Q}]\{\varepsilon\}_{xy}$)

3.2.6. Hygrothermal Effects in a Laminate

In Section 3.2.3.1, the hygrothermal strains were calculated for an angle and unidirectional lamina subjected to a temperature change, ΔT, and moisture content change, ΔC. As mentioned previously, if the lamina is free to expand, no residual mechanical stresses would develop in the lamina at the macromechanical level. However, in a laminate with various plies of different angles or materials, each individual lamina is not free to deform. This results is residual stresses in the laminate.

3.2.6.1. Hygrothermal Stresses and Strains

Sources of hygrothermal loads include cooling down from processing temperatures, operating temperatures different from processing temperatures, and humid environments such as in aircraft flying at high altitudes. Each ply in a laminate gets stressed by the deformation differences of adjacent lamina. Only the strains in excess of or less than the hygrothermal strains in the unrestricted lamina produce the residual stresses. These strain differences are called mechanical strains and the stresses caused by them are called mechanical stresses.
The total strains induced by hygrothermal and mechanical loads are;

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}{\varepsilon_x}^M\\ {\varepsilon_y}^M\\ {\gamma_{xy}}^M\end{Bmatrix}+\begin{Bmatrix}{\varepsilon_x}^T\\ {\varepsilon_y}^T\\ {\gamma_{xy}}^T\end{Bmatrix}+\begin{Bmatrix}{\varepsilon_x}^H\\ {\varepsilon_y}^H\\ {\gamma_{xy}}^H\end{Bmatrix} \tag{3.59}$$

Using stress–strain Equation (3.26), the hygrothermal stresses in a lamina are then given by;

$$\begin{Bmatrix}{\sigma_x}^{TH}\\ {\sigma_y}^{TH}\\ {\tau_{xy}}^{TH}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{Bmatrix}{\varepsilon_x}^M\\ {\varepsilon_y}^M\\ {\gamma_{xy}}^M\end{Bmatrix} \tag{3.60}$$

Where TH stands for combined thermal and moisture effects. Hygrothermal stresses induce zero resultant forces and moments in the laminate.

$$\int_{-z/2}^{z/2}\begin{Bmatrix}{\sigma_x}^{TH}\\ {\sigma_y}^{TH}\\ {\tau_{xy}}^{TH}\end{Bmatrix}zdz=0=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}{\sigma_x}^{TH}\\ {\sigma_y}^{TH}\\ {\tau_{xy}}^{TH}\end{Bmatrix}zdz \tag{3.61}$$

From Equation (3.60) to Equation (3.61);

$$\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{Bmatrix}{\varepsilon_x}^M\\ {\varepsilon_y}^M\\ {\gamma_{xy}}^M\end{Bmatrix}zdz \tag{3.62}$$

Substituting Equation (3.59) and Equation (3.45) in Equation (3.62) gives;

$$\begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+\begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix}=\begin{Bmatrix}{N_x}^T\\ {N_y}^T\\ {N_{xy}}^T\end{Bmatrix}+\begin{Bmatrix}{N_x}^H\\ {N_y}^H\\ {N_{xy}}^H\end{Bmatrix} \tag{3.63}$$

$$\begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+\begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix}=\begin{Bmatrix}{M_x}^T\\ {M_y}^T\\ {M_{xy}}^T\end{Bmatrix}+\begin{Bmatrix}{M_x}^H\\ {M_y}^H\\ {M_{xy}}^H\end{Bmatrix} \tag{3.64}$$

The four arrays on the right-hand sides of Equation (3.63) and Equation (3.64) are given by;

$$\{N^T\} = \begin{Bmatrix} {N_x}^T \\ {N_y}^T \\ {N_{xy}}^T \end{Bmatrix} = \Delta T \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k (z_{k+1} - z_k) \tag{3.65}$$

$$\{M^T\} = \begin{Bmatrix} {M_x}^T \\ {M_y}^T \\ {M_{xy}}^T \end{Bmatrix} = \frac{1}{2}\Delta T \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k ({z_{k+1}}^2 - {z_k}^2) \tag{3.66}$$

$$\{N^H\} = \begin{Bmatrix} {N_x}^H \\ {N_y}^H \\ {N_{xy}}^H \end{Bmatrix} = \Delta C \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k (z_{k+1} - z_k) \tag{3.67}$$

$$\{M^H\} = \begin{Bmatrix} {M_x}^H \\ {M_y}^H \\ {M_{xy}}^H \end{Bmatrix} = \frac{1}{2}\Delta C \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k ({z_{k+1}}^2 - {z_k}^2) \tag{3.68}$$

Then;

$$\{N\} + \{N^T\} + \{N^H\} = [A]\{\varepsilon^0\} + [B]\{\kappa\} \tag{3.69}$$

$$\{M\} + \{M^T\} + \{M^H\} = [B]\{\varepsilon^0\} + [D]\{\kappa\} \tag{3.70}$$

Let $\{N^E\}$ and $\{M^E\}$ be the total, effective forces and moments. So;

$$\{N^E\} = \{N\} + \{N^T\} + \{N^H\} \tag{3.71}$$

$$\{M^E\} = \{M\} + \{M^T\} + \{M^H\} \tag{3.72}$$

Then;

$$\begin{Bmatrix} N^E \\ M^E \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{3.73}$$

And;

$$\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} = \begin{bmatrix} [\alpha] & [\beta] \\ [\beta^T] & [\delta] \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \tag{3.74}$$

3.2.7. Stacking Sequence Notation

The notation which is used in this thesis is described by the help of the figure illustrated below.

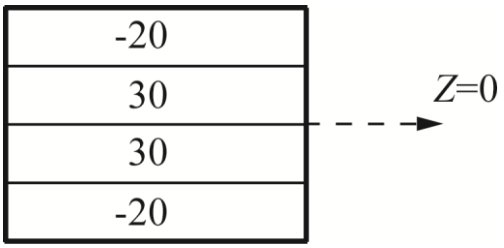


Figure 3.14. Stacking Sequence of a Laminate

A laminate is made of a group of laminae, represented by the angle of ply and separated from other plies by a comma, bonded to each other. Starting from the bottom lamina, $[-20, 30, 30, -20]_T$ denotes the code for the above laminate consisting of four plies in which the subscript T stands for a total laminate. A comma separates each lamina. Also $[-20, 30]_S$ can be used for this laminate since the plies above the midplane are of the same orientation, material, and thickness as the plies below the midplane which makes this a symmetric laminate. The subscript S stands for "Symmetric". Sometimes $[-20, 30_2, -20]_T$ may also be used to denote the same laminate. Hence two 30° plies are adjacent to each other, 30_2 denotes them, where the subscript 2 is the number of adjacent plies of the same angle.

3.2.8. Composite Failure Theories

The composite laminate fails when any one of the laminae fails. This is called the first-ply failure (FPF). It is possible that some layer fails first and that the composite continues to take more loads until all the plies fail. Failed plies may

still contribute to the stiffness and strength of the laminate. In order to describe Failure Criterion, firstly on-axis failure stresses for a lamina should be defined;

$(\sigma_1)_{max}$ in tension : X $(\sigma_1)_{max}$ in compression : X'

$(\sigma_2)_{max}$ in tension : Y $(\sigma_2)_{max}$ in compression : Y'

$(\tau_{12})_{max}$ in shear : S

Table 3.2 Failure Stresses Properties of Some Composite Material (MPa)

Material	V^f	X	X'	Y	Y'	S
Graphite/Epoxy	0.7	1500	1500	40	246	68
Aramid/Epoxy	0.6	1400	235	12	53	34
Glass/Epoxy	0.45	1062	610	31	118	72

Unlike the stiffness parameters, these strength parameters cannot be transformed directly for an angle lamina. Thus, the failure theories are based on first finding the stresses in the local axes and then using these five strength parameters of a unidirectional lamina to find whether a lamina has failed. Four common failure theories are given as:

a) *Maximum Stress Criterion*: No interaction of stresses is considered.
b) *Maximum Strain Criterion:* No interaction of stresses is considered.
c) Tsai-Hill Criterion: Considers combine effects of stresses but assumed tension strengths are equal to compression strengths.
d) Tsai-Wu Criterion: Accounts for combined effects and differences in tensile and compressive strengths.

The most commonly-used failure theory is Tsai-Wu theory. This theory is comparable to the Von Mises theory used in isotropic material analysis and it will be used in the present work.

According to Tsai-Wu Criterion, the material (lamina) is safe as long as;

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}{\sigma_1}^2 + F_{22}{\sigma_2}^2 + F_{66}{\tau_{12}}^2 + 2F_{12}\sigma_1\sigma_2 \leq 1 \quad (3.75)$$

Where σ_i are on axis stresses and;

$$F_1 = \frac{1}{X} - \frac{1}{X'}$$
$$F_{11} = \frac{1}{XX'}$$
$$F_1 = \frac{1}{Y} - \frac{1}{Y'}$$
$$F_{22} = \frac{1}{YY'} \quad (3.76a\text{-}f)$$
$$F_{66} = \frac{1}{S^2}$$
$$F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}}$$

3.2.8.1. Reserve Factor

Given the on axis stresses $\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}$ in a ply, failure stress state is defined as $\begin{Bmatrix}{\sigma_1}'\\ {\sigma_2}'\\ {\tau_{12}}'\end{Bmatrix} = \begin{Bmatrix}R\sigma_1\\ R\sigma_2\\ R\tau_{12}\end{Bmatrix}$ where R is reserve factor (similar to the Factor of Safety used in elementary mechanics of materials). If $R\sigma_i$ is put into Equation (3.75) and terms are collected in R for failure state;

$$(F_{11}{\sigma_1}^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}{\sigma_2}^2 + F_{66}{\tau_{12}}^2)R^2 + (F_1\sigma_1 + F_2\sigma_2)R - 1 = 0 \quad (3.77)$$

Letting

$$(F_{11}{\sigma_1}^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}{\sigma_2}^2 + F_{66}{\tau_{12}}^2) = a$$
$$(F_1\sigma_1 + F_2\sigma_2) = b \quad (3.78a\text{-}c)$$
$$1 = c$$

We get,

$$aR^2 + bR - c = 0 \tag{3.79}$$

If Equation (3.79) is solved for R;

$$R = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$$

$$R_1 = \frac{-b + \sqrt{b^2 + 4ac}}{2a} \tag{3.80a-c}$$

$$R_2 = \frac{-b - \sqrt{b^2 + 4ac}}{2a}$$

R has two roots, one is positive (R_1) and the other is negative (R_2). For safety $|R|$ must be greater than one.

Positive R means that the applied stress can be increased by a factor of R_1 and negative R means that the applied stress can be reversed by a factor of R_2

4. RESULTS AND DISCUSSIONS

In this study, the objective is to create an efficient and user-friendly computer program for the purpose of analyzing static problems of fiber-reinforced composites. KomPro; a Matlab GUI-based executable composite mechanics software is presented to the Turkish researchers and academicians.

In order to describe how to install and use the software on computers, a comprehensive and easy-to-follow user manual is prepared for the user.

4.1. User Manual of KomPro

The installation procedure, how to input the data and calculation steps are described in detail with visual examples.

4.1.1. Introduction to KomPro

Engineers, scientists and students who are interested in mechanics of composites are faced with comprehensive and iterative algebraic operations such as matrix multiplication, inversion and transposition of matrices, etc. In the design of laminated structures, writing a computer program or using a mathematical package such as Matlab and Mathematica becomes absolutely essential. However, building a professional and efficient program for all the principles of mechanics of composites may be time consuming. At that point, engineers, scientists and students can use KomPro, the tool of computational composite mechanics.

4.1.2. Product Description

KomPro is the Matlab GUI based user-friendly software package that can solve advanced static problems of mechanics of composite plates and provide ease of use, saving of time, and interface quality. Using KomPro, user can design and analyse composite plates. Beside the mechanical effects, user can take into

consideration the situation at which hygrothermal effects present. KomPro provides the four different solution forms as strain-curvature, strain-moment, load-moment, and load-curvature. Through the instrument of KomPro, user can perform failure analysis on each ply of designed plate using the Tsai-Wu tensor failure criterion.

4.1.3. Installing KomPro

If someone who wants to set up KomPro, he should follow the steps below.

I. Save the 'KomPro folder' to desktop of the computer or directory of your choice

II. Open *KomPro* and click the *distrib* folder

III. Run the *KomPro_pkg.exe* and replace the *_install.bat, MCRInstaller.exe, KomPro.exe* and *readme.txt* files

IV. Choose the language and install the *Matlab Compiler Runtime*

V. After installation of the compiler, run the *KomPro* by clicking *KomPro.exe* in the *distrib* folder.

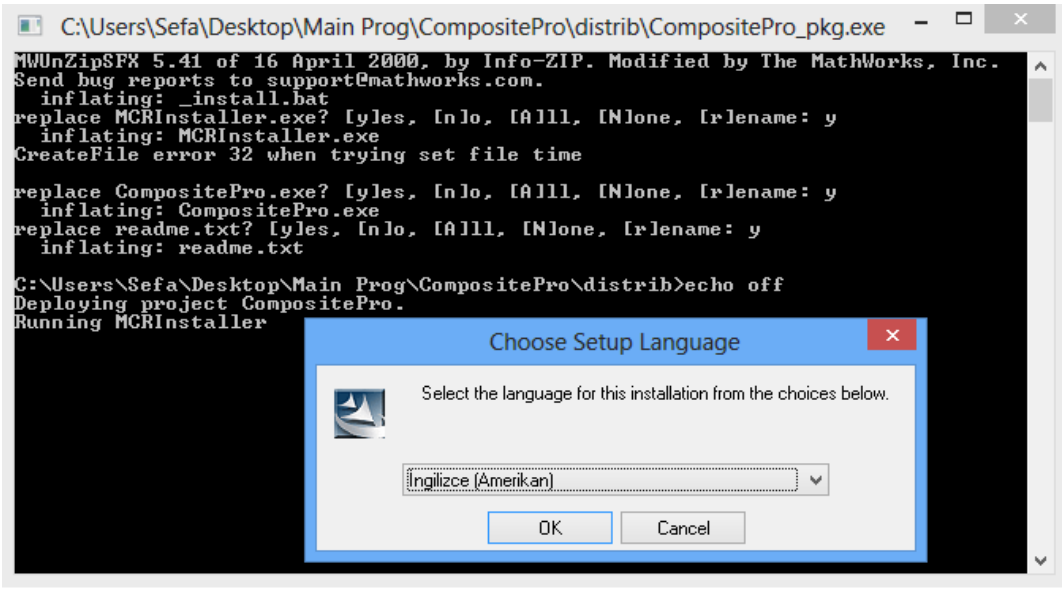


Figure 4.1. Installation Steps of KomPro

4.1.4. Interfaces

KomPro consists of a main and a sub interfaces which are called ‘Data Input’ and ‘Calculations’ respectively.

4.1.4.1. Main (Data Input) Interface

In the main interface which is be opened after performing the KomPro, user can determine the number of plies and materials of the plate to be designed. Also in this section, user can decide whether he carries out failure analysis and the laminate is symmetric or not. Once user has defined the number of plies and materials, material properties and ply thicknesses can be entered into the panel that will be opened after clicking the submit button of Number of Material section or selected from pre-defined material library. Moreover, stacking sequence convention for laminates can be set down and the angle of fiber orientation can be designated later on passing the steps mentioned before.

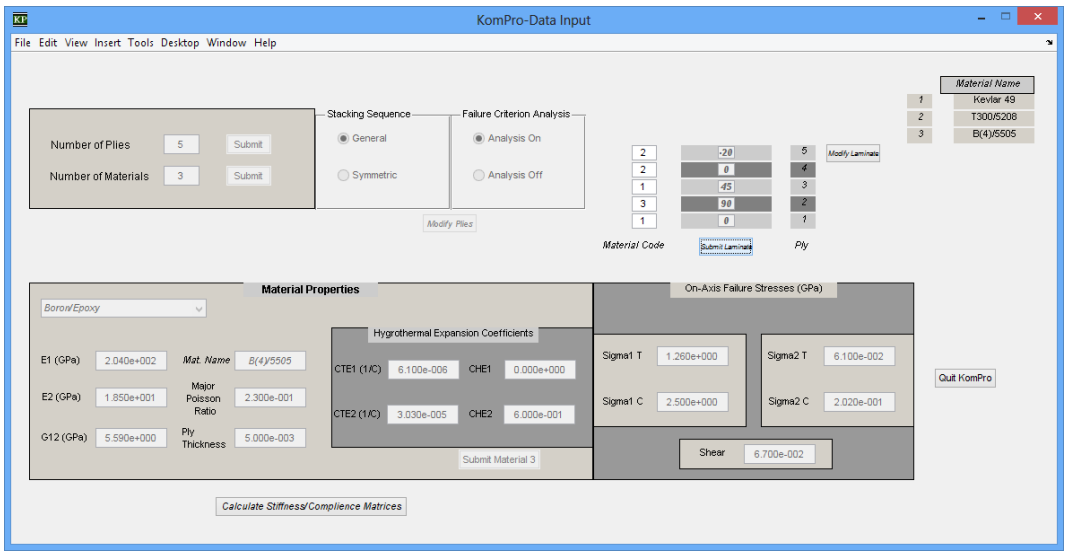


Figure 4.2. The Main Interface of KomPro

4.1.4.1.(1). Material Library

Material properties and thickness of four commonly used composite lamina as Graphite/Epoxy, Kevlar/Epoxy, Boron/Epoxy, Glass/Epoxy and two

isotropic metals as aluminum and steel can be chosen from material database of KomPro. The user can also display and edit the values of material properties and thickness of these materials.

In material properties menu illustrated below, user can input fifteen elements of the lamina to be used for the stacking sequences of laminate. These are the 4 elastic moduli, ply thickness, the 5 strength parameters, the 4 coefficients of thermal and moisture expansion, and lamina name of a specially orthotropic lamina.

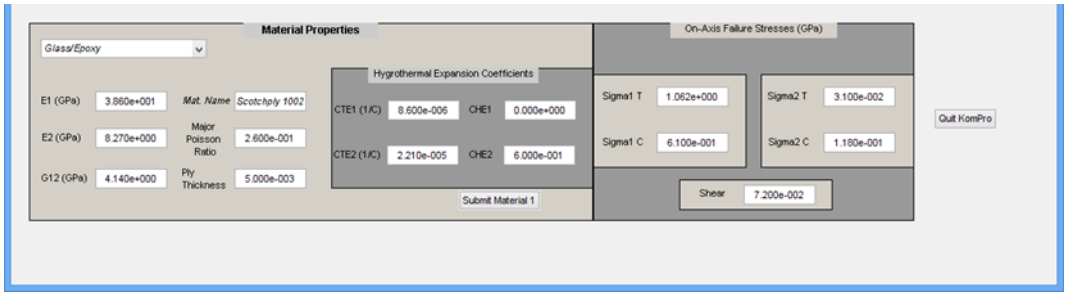


Figure 4.3. Material Properties Menu of KomPro

4.1.4.2. Sub (Calculations) Interface

In the sub interface opened after clicking the 'Calculate Stiffness/Compliance Matrices' button, KomPro displays stiffness and compliance matrices of designed composite plate and in plane, coupling, and bending stiffness and compliance terms can be seen separately. User will also select solution forms after determining the type of applied effects from selection menu bars.

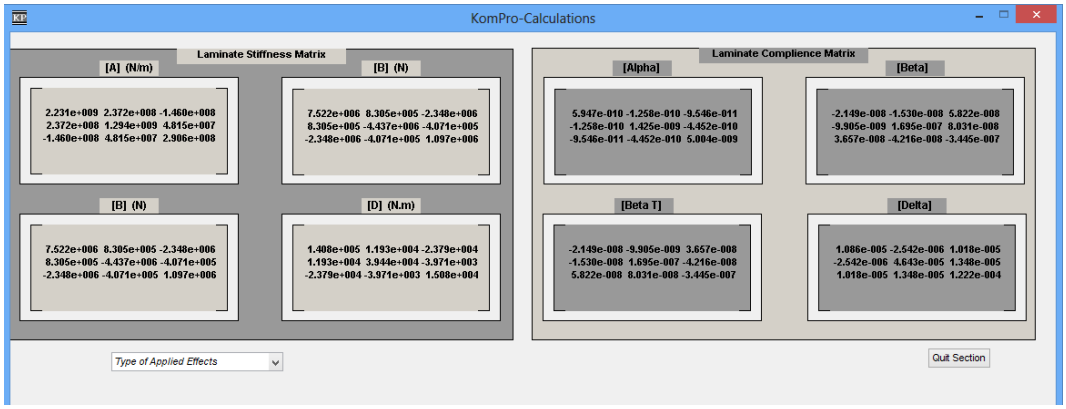


Figure 4.4. Stiffness/Compliance Matrices and Selection Menu Bars Section of Sub Interface

4.1.5. Displaying the Results

After defining the solution form and type of applied effect and inputting the data, desired results will be displayed by clicking the related push button. If failure criterion was selected, result of failure analysis may also be displayed in the result section illustrated in Figure 4.5.

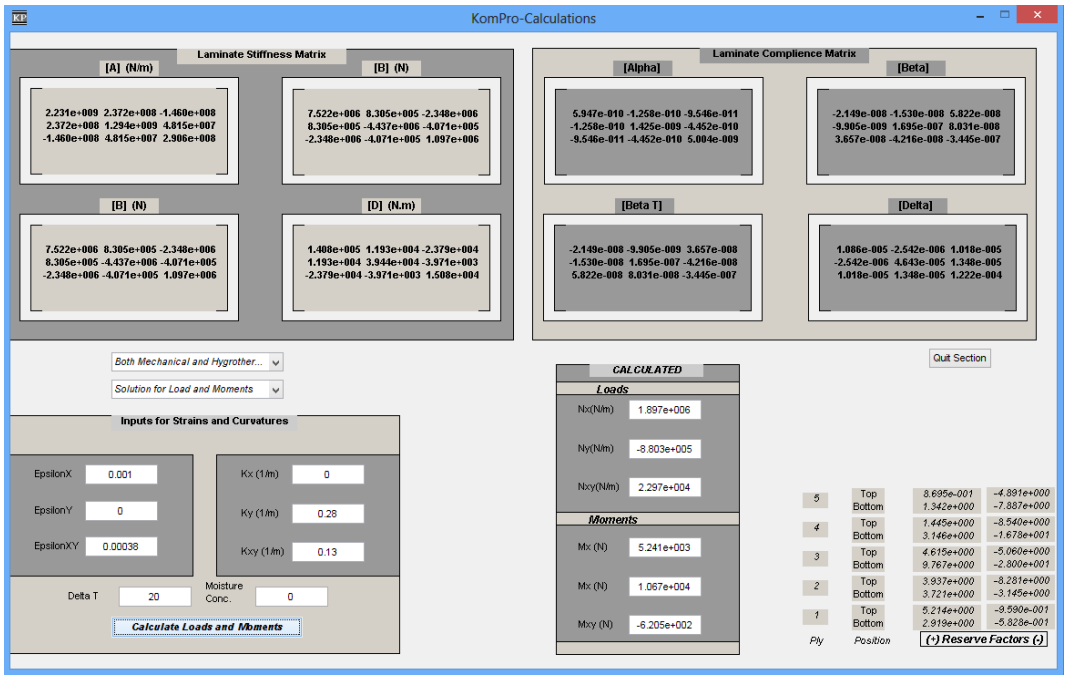


Figure 4.5. The Sub Interface of KomPro with Result Section

4.2. Example

An extended example taken from (Kaw, 2005) is solved in order to show in detail how to analyze a fiber-reinforced composite plate by KomPro.

For a $[0,90,0]$ graphite/ epoxy laminate, the only load applied is a unit tensile normal load in the x-direction — that is, the direction parallel to the fibers in the 0° ply and assuming the thickness of each ply is 5 mm;

a) Find the stiffness and compliance terms for the laminate
b) Calculate midplane strain and curvatures
c) Find the reserve factors for the bottom and top surface of all plies
d) Perform a, b and c taking into account temperature differences and moisture intake together with tensile normal load. Considering the laminate is cured at 176.7 ℃ and tested at 21.1 ℃ and the amount of moisture concentration absorb by the composite is 0.007 g/g.

4.2.1. Solution of Example Using the Software

First of all number of plies and number of materials for this problem should be input on the related section of the software which is illustrated below. The laminate has three plies and consists of only one material, graphite/epoxy.

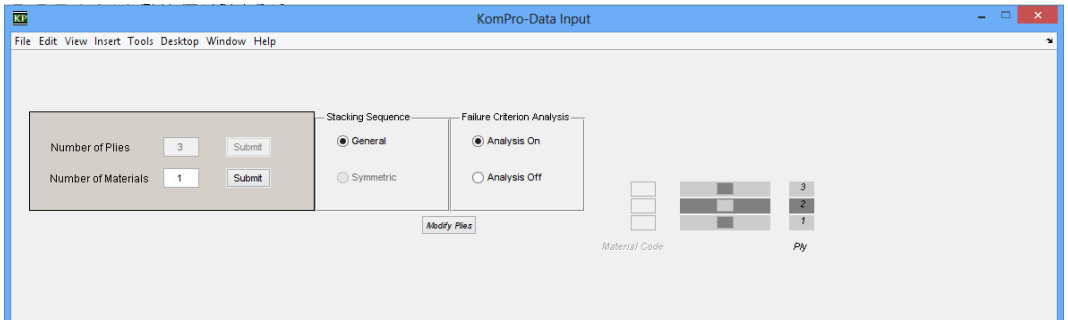


Figure 4.6. Inputting the Number of Plies and Materials on KomPro

Also ‘Failure Analysis On’ button should be marked if reserve factors mentioned at b are wanted. After clicking submit button of number of materials,

material properties of graphite/epoxy can be selected from the material library. This step is shown in Figure 4.7.

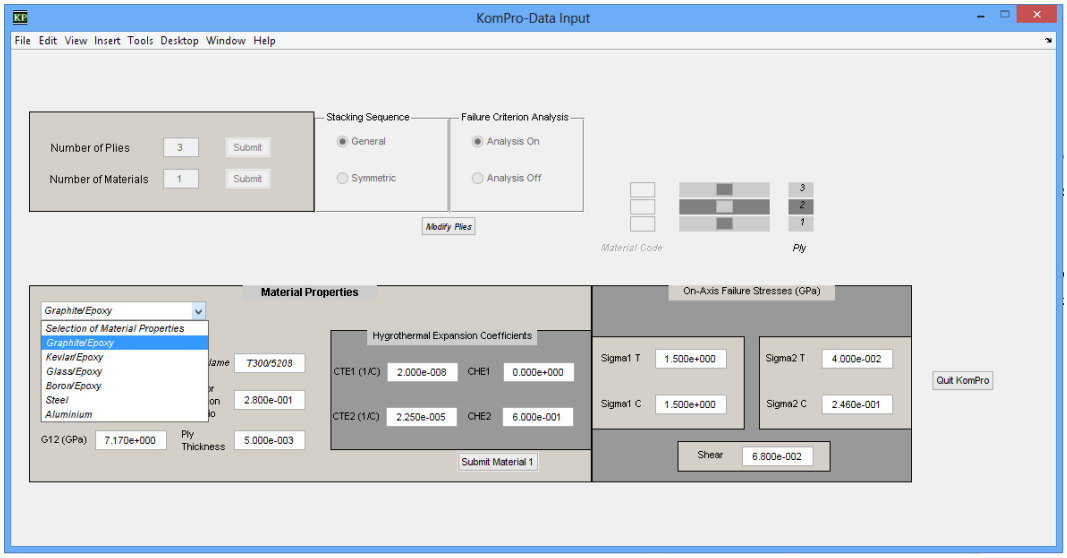


Figure 4.7. Selection of Material Properties from Material Library

As illustrated in Figure 4.8, after selecting material properties of graphite/epoxy, fiber angle of each ply are entered for $[0,90,0]_T$ laminate. Material codes of plies are automatically designated as one by the software because the laminate is formed by only one material.

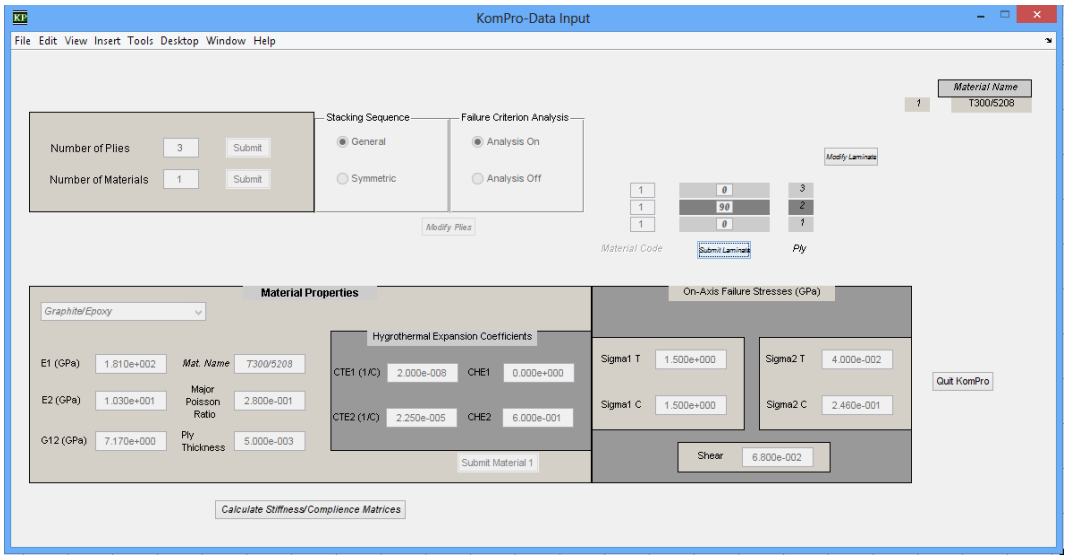


Figure 4.8. Defining Stacking Sequence and Inputting Material Properties of $[0,90,0]_T$ Graphite/Epoxy Laminate

Stacking sequence has defined, then stiffness and compliance terms for the laminate mentioned in the first article of the question can be found by clicking related push button and shown in Figure 4.9.

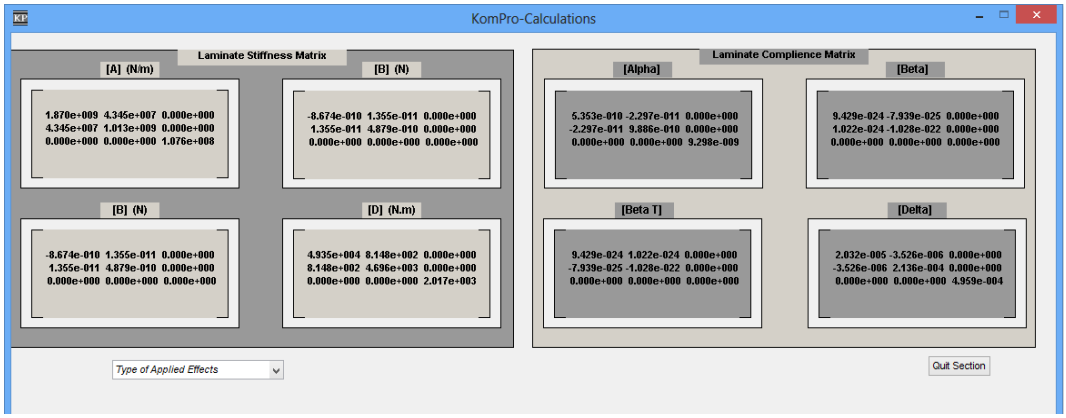


Figure 4.9. The Stiffness and Compliance Terms for the Laminate

For the second article of the question, the type of applied effects is selected ‘Mechanical Effects Only’ and the solution form is chosen ‘Solution for Strains and Curvatures’ to calculate midplane strain and curvatures as shown below. Then midplane strains and curvatures are calculated for the laminate subjected to only tensile normal load in the x-direction by clicking ‘Calculate Strain and Curvature’ push button. Also the reserve factors for the bottom and top surface of all plies are computed since the failure criterion analysis is already selected in the main interface. Results of article b and c are illustrated in Figure 4.10.

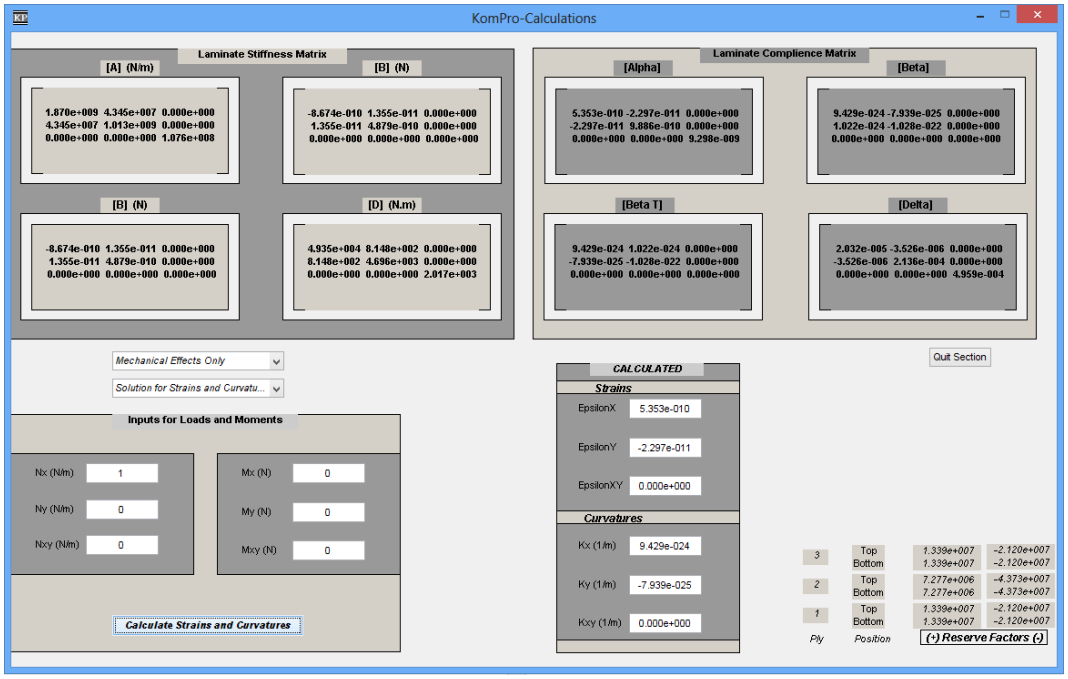


Figure 4.10. The Midplane Strains and Curvatures of the Laminate and Reserve Factors for the Bottom and Top Surface of All Plies

Here only $N_x = 1\ N/m$ is present other all N_i and M_i are zero hence the laminate is only subjected to normal load in x-direction. Also the midplane curvatures are zero because the laminate is symmetric and no bending and no twisting loads are applied.

For the last article, while the solution form remains same, the applied effect is changed to 'Both Mechanical and Hygrothermal Effects' due to the temperature differences and moisture effect. Also neither laminate stiffness matrix nor compliance matrix are different since there are no change in the material properties or stacking sequences of laminate. Here the temperature difference is -155.6 ℃ and moisture concentration is 0.007 g/g. When inputted into KomPro, the results are displayed on the sub interface of the software shown in Figure 4.11.

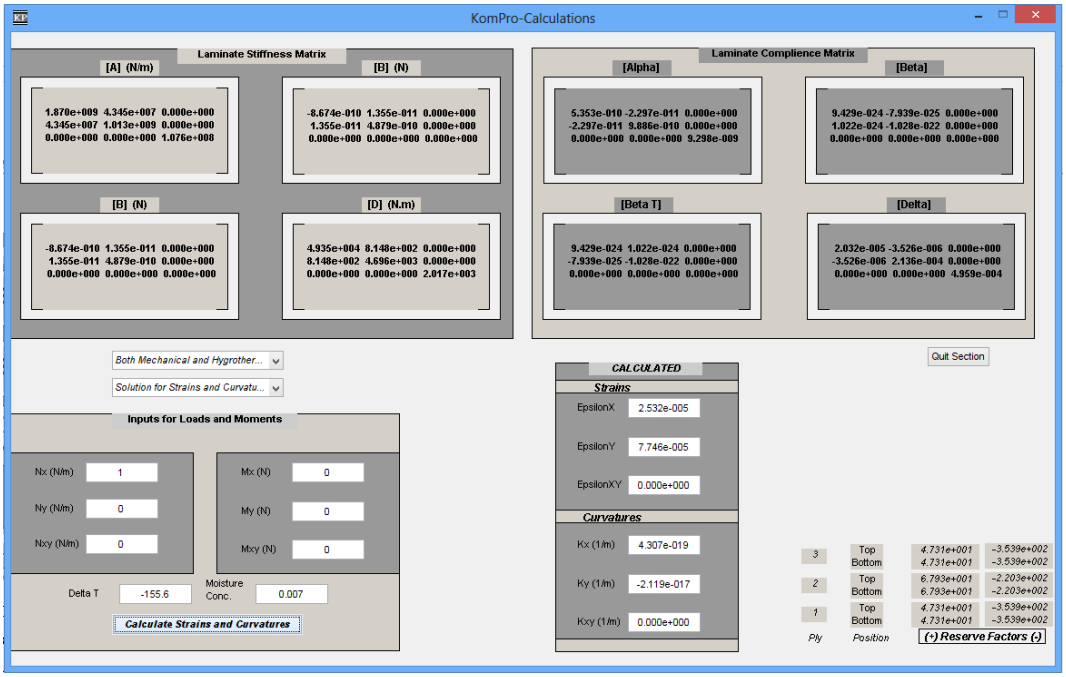


Figure 4.11. The Midplane Strains and Curvatures of the Laminate and Reserve Factors for the Bottom and Top Surface of All Plies Taking into Account Hygrothermal Effects

5. CONCLUSIONS

In this study a computer program called KomPro was designated by means of Matlab GUI. KomPro is a user-friendly software package with its push, radio buttons, and popup menus that provide ease of use while inputting material properties and selecting applied effects etc. KomPro is faster considering the other academic-purposed software and it is easily-accessible compared to the other commercial programs.

A material library in which the properties of frequently-used materials in composite structures are included is placed in KomPro in order to make finding materials an easy and convenient task for the users. There is an interchange button when it is needed to change the composite plies as the user wants to design or analyse. After defining stacking sequences of the composite plate, stiffness and compliance terms of plate can be displayed. Besides analysing the mechanical influences and hygrothermal influences separately on the composite plate, the user also take into consideration the situations at which both mechanical and hygrothermal influences are present together while analysing composite plate using KomPro. Together with these influences, the software offers different solution sets such as strain-curvature, strain-moment, load-moment and load-curvature. KomPro also involves the option of performing failure analysis on each ply using the Tsai-Wu tensor failure criterion. In addition to all of these, it can be used as an executable form and may be run in computers without the need for installation of Matlab.

This software which implements only in-plane static analysis as a part of a Master thesis work may be considered as the first step for a Ph.D. study and will form a basis for the design of comprehensive software package which can tackle both static, dynamic and stability problems.

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