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SCATTERING OF PLANE WAVES BY CORRUGATED SURFACES

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ABSTRACT

In this study, scattering of plane electromagnetic waves at the junction of two corrugated half-planes is investigated. It has been shown that the scattering problem can be simulated with anisotropic impedance half-planes. By formulating with Fourier integral transform technique, the problem is reduced to a Matrix Wiener-Hopf system which is accomplished by via Daniele-Khrapkov method. Alternatively, due to the particular form of the matrix Wiener-Hopf problem, by successive elimination of the unknown functions it has been solved as a scalar Wiener-Hopf problem also. The diffracted field is expressed in a form suitable for GTD applications and the behaviour of the surface waves are also investigated.

ÖZ

Bu çalışmada, farklı periyotlu oluklu iki yarı düzlem ekleminden düzlem dalgaların saçılma mekanizması incelenmiştir. Fourier İntegral Transformu tekniği ile saçılma problemi Matris Wiener-Hopf denklem sistemine indirgenmiştir. Daniele-Khrapkov metodu ile Matris Wiener-Hopf denklem sistemi çözülmüş ve kırınan alan ifadesi elde edilmiştir. Buna ilaveten, yapı boyunca oluşabilecek yüzey dalgaları incelenmiş ve yüzey dalgalarının hızının yapının parametrelerine bağımlılığı incelenmiştir. Ayrıca, gözönüne alınan Matris Wiener-Hopf denkleminin özel durumu nedeniyle, bilinmeyen fonksiyonlar ardışık eliminasyonu yöntemi ile birbiri cinsinden ifade edilebilmiş ve skaler Wiener-Hopf problemi olarak çözülüştür.

1. INTRODUCTION

Corrugated structures are widely used especially in waveguides and antenna applications. As a slow-wave structure, they can be applied in waveguides to use their guiding properties. Although there have been many works which considered them as slow-wave structures, some of the early works have also been performed for their radiation characteristics. One of the attractive studies carried out by Reynolds and Lucke [1] has given the radiating characteristics of the corrugated surfaces. They have formed a high gain travelling wave antenna using radiation properties of a corrugated metallic surface embedded in ground plane. Another work which is related with the scanning surface wave antennas using the above structure is due to Hougardy and Hansen [2]; where the existence of a surface wave was shown which propagates across a corrugated metallic surface at an oblique angle. It has been shown that the surface wave velocity increases smoothly to that of free space when the wave direction approaches to be parallel to the corrugations. Bhattacharyya [3] has designed a low backscattering surface using this kind of a structure as a solution to the problem of echos from terminal building and large aircraft hangers in crowded airports. Scattering mechanism of a H-polarized plane wave from a grating composed of narrow grooves in a perfectly conducting surface has been investigated by Hurd and Jull [4] using conformal transformation techniques where it has been assumed that the ratio of the slot width to the grating period is small. Another important study given by Bryant [5] has presented the theoretical analysis of a square cross-section corrugated waveguide and it has been shown that both fast and slow waves occur in a square waveguide with all four walls corrugated. Bryant has used anisotropic impedance characteristics of a corrugated wall while formulating the problem.

There are similarities between waves propagating on a corrugated infinite plane, and those in a waveguide with one wall corrugated [6]. Hurd [7] has investigated the propagation of surface waves along an infinite corrugated surface. To facilitate the solution of the problem, he has assumed an idealized physical structure on which the corrugations are vanishingly thin and the slot width is small compared with both the slot height and the free space wavelength. The method employed by Hurd involves writing the complete fields both above the corrugations and in the slots, matching the tangential field components at the boundaries and then using the calculus of residues technique to evaluate the resulting infinite set of simultaneous equations.

It has been known for a long time that impedance boundary conditions can be employed to simulate corrugated surfaces with infinitely thin boundaries. Weinstein [8] has given the boundary conditions of second order for a corrugated surface which reduces to the standard impedance boundary conditions in the case of a first order approximation. These conditions show the anisotropic character of the corrugated structure explicitly. In this work, the boundary conditions given by Weinstein are employed to simulate two corrugated half-planes with same slot height but different slot width. The scattering mechanism at the junction of these half-planes is investigated, which may correspond to the discontinuity when two corrugated waveguides with different corrugations are joined. First, above-mentioned boundary conditions are summarized and the formulation of the related boundary-value problem via Fourier transform technique is given. Then, the problem is reduced to the solution of a matrix Wiener-Hopf equation which is accomplished via Khrapkov-Daniele method. The results are presented both for the surface wave behaviour at the junction and for the diffracted field dependence on geometrical parameters of the half-planes. It is shown that surface waves which are determined from the residue contributions of the scattered field exist in both half-planes depending upon the impedance of each half-plane. Especially, the effect of the slot width on the surface wave velocity is very important since such a structure may behave like surface wave-passband or stopband.

2. APPROXIMATE BOUNDARY CONDITIONS

For the corrugated structure shown in Fig.-1a, it has been assumed that $t \ll 1$, $\mathrm{w} \ll \lambda$ and $\mathrm{w} \ll h$, with the structure being extended to infinity from all sides to cover the whole plane. This corresponds to an infinite strip grating placed over an infinite perfectly conducting substrate. As is seen from Fig.-1, the parameters t, w, and h denote, respectively, the thickness of the corrugations, the slot width and the slot height. In order to simulate such a structure with an infinitely thin fictitious surface situated at $y = 0$, Weinstein [8] has introduced the following boundary conditions:

$$E_z(x,0,z) = ikhZ_o H_x(x,0,z) + (h-q)\frac{\partial}{\partial z}E_y(x,0,z) \tag{1}$$

$$E_x(x,0,z) = -ik(h-q)Z_o H_z(x,0,z) + (h-q)\frac{\partial}{\partial x}E_y(x,0,z) \tag{2}$$

with

$$q = (\mathrm{w}/\pi)\ln[\cosh(\pi h/\mathrm{w})]\,, \tag{3}$$

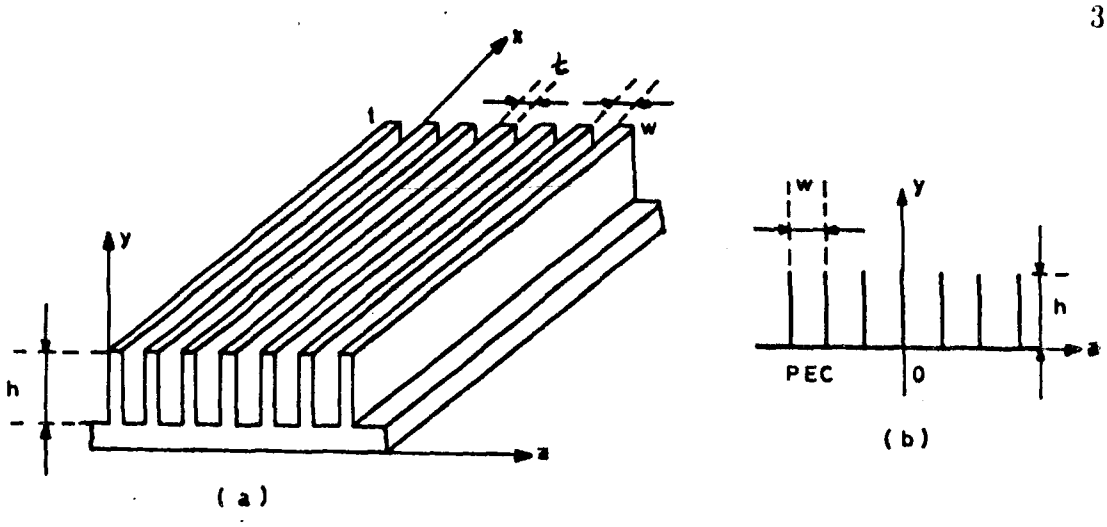


Figure-1. Illustration of a corrugated surface : (a) real structure with material thickness, (b) idealized structure with vanishingly thin corrugations.

which are valid for the case when $kh \ll 1$ and $kw \ll 1$. They show the anisotropic character of the structure explicitly and give the dependence of the surface impedance values to the corrugated surface parameters. Here, $Z_o = \sqrt{\mu/\varepsilon}$ is the characteristic impedance of the medium with ε and μ denoting the dielectric permittivity and magnetic permeability, respectively.

First order approximation of these conditions can be obtained by eliminating the higher order derivatives:

$$\begin{bmatrix} E_x \\ E_z \end{bmatrix} = Z_o \begin{bmatrix} 0 & -ik(h-q) \\ ikh & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_z \end{bmatrix}. \tag{4}$$

As is known, these are the standard impedance boundry conditions corresponding to an anisotropic impedance surface.
The impedance boundary condition which is usually attributed to Leontovich has been introduced almost four decades ago. It is usually expressed as

$$\hat{n} \times \hat{n} \times \vec{E} = -Z_o \bar{\bar{Z}} \hat{n} \times \vec{H} \tag{5}$$

where $\hat{n}$ is the outward unit normal vector to the surface. In the most general form, for a surface located on the plane $y = 0$, the relative impedance can be written in dyadic form as

$$\bar{\bar{Z}} = Z_{xx}\hat{e}_x\hat{e}_x + Z_{xz}\hat{e}_x\hat{e}_z + Z_{zx}\hat{e}_z\hat{e}_x + Z_{zz}\hat{e}_z\hat{e}_z \tag{6}$$

where $\hat{e}_x$ and $\hat{e}_z$ denote the tangential unit normal vectors in the Cartesian coordinate system. In terms of the field components, these conditions can be written as follows with matrix form

$$\begin{bmatrix} E_x \\ E_z \end{bmatrix} = Z_o \begin{bmatrix} -Z_{xz} & Z_{xx} \\ -Z_{zz} & Z_{zx} \end{bmatrix} \begin{bmatrix} H_x \\ H_z \end{bmatrix}. \tag{7}$$

By comparing the general anisotropic impedance boundary conditions with the conditions given by Weinstein for simulating a corrugated surface, it can easily be seen that $Z_{zx} = Z_{xz} = 0$. The structure which is considered in this work involves two corrugated half planes with same corrugation thickness and slot height but different slot widths as shown in Fig.-2a. If these points are taken into consideration it is obvious that on the half-planes different impedance values will be written in the z-direction and same impedance will be imposed in the x-direction. The relative surface impedance values in terms of the corrugated structure parameters are (see Fig.-2b) :

$$Z_z^+ = -ik(h - q^+) \quad ; \qquad Z_z^- = -ik(h - q^-) \tag{8}$$

and

$$Z_x^+ = Z_x^- = Z = -ikh \tag{9}$$

with

$$q^+ = \frac{w_1}{\pi} \ln[\cosh \frac{\pi h}{w_1}] \quad \text{and} \quad q^- = \frac{w_2}{\pi} \ln[\cosh \frac{\pi h}{w_2}]. \tag{10}$$

So, the boundary conditions to be imposed on the Oxz plane can be expressed as:

$$\begin{bmatrix} E_x(x,0,z) \\ E_z(x,0,z) \end{bmatrix} = Z_o \begin{bmatrix} 0 & Z_z^+ \\ -Z & 0 \end{bmatrix} \begin{bmatrix} H_x(x,0,z) \\ H_z(x,0,z) \end{bmatrix}, \quad \text{for } z > 0 \tag{11}$$

and

$$\begin{bmatrix} E_x(x,0,z) \\ E_z(x,0,z) \end{bmatrix} = Z_o \begin{bmatrix} 0 & Z_z^- \\ -Z & 0 \end{bmatrix} \begin{bmatrix} H_x(x,0,z) \\ H_z(x,0,z) \end{bmatrix}, \quad \text{for } z < 0. \tag{12}$$

It should be noted that a similar configuration has been recently considered by Serbest et al. [9] which has motivated the present work. The structure in [9] also involves the discontinuity of two anisotropic impedance half-planes with assuming that the half-planes are perfectly conducting in the direction parallel to the edge and have different impedances in the direction perpendicular to the edge. This case

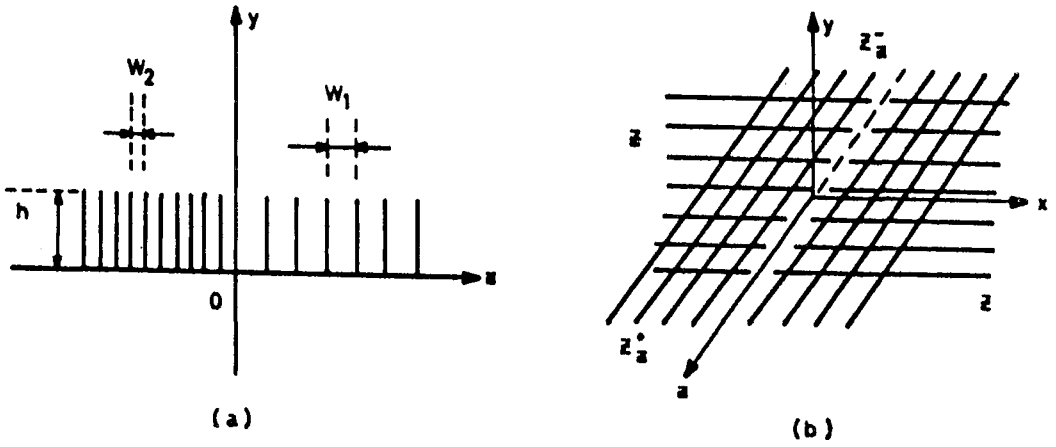


Figure-2. Geometry of the problem : (a) Junction formed by two corrugated half-planes with different corrugation widths, (b) Discontinuity formed by two anisotropic impedance half-planes.

corresponds to writing $Z = 0$ in (11-12) which implies $h = 0$ and this corresponds to a smooth surface without corrugations.

3. FORMULATION OF THE PROBLEM

The anisotropic impedance surface is illuminated from $y > 0$ region by a plane wave at oblique incidence :

$$\vec{E}^i(x,y,z) = \vec{A}^i(x,y,z)\,\exp\{-i\vec{k}\cdot\vec{r}\} \tag{13}$$

where

$$\vec{A}^i = A^i_x\,\hat{x} + A^i_y\,\hat{y} + A^i_z\,\hat{z} \tag{14}$$

and

$$\vec{k} = k_x\,\hat{x} + k_y\,\hat{y} + k_z\,\hat{z} \tag{15}$$

with

$$\vec{k}.\vec{A}^i = 0 \qquad \text{and,} \qquad k_x = k\sin\phi_o\cos\theta_o\,, \quad k_y = k\sin\phi_o\sin\theta_o\,, \quad k_z = k\cos\phi_o\,. \tag{16}$$

The incident magnetic field can easily be expressed as follows in terms of the incident electric field:

$$\begin{bmatrix} H_x^i \\ H_y^i \\ H_z^i \end{bmatrix} = (1/\omega\mu) \begin{bmatrix} 0 & -k_z & -k_y \\ k_z & 0 & k_x \\ k_y & -k_x & 0 \end{bmatrix} \begin{bmatrix} E_x^i \\ E_y^i \\ E_z^i \end{bmatrix}. \tag{17}$$

Here k is the wave number of the surrounding medium and for analytical convenience it is assumed that the wave number has a small positive imaginary part. The x-dependence $\exp(-ik_x x)$ and the time dependence $\exp(-i\omega t)$, which are common to all field quantities, will be suppressed throughout the analysis.
Let us split up the total field as for all y

$$\vec{E}(\vec{H}) = \vec{E}^i(\vec{H}^i) + \vec{E}^r(\vec{H}^r) + \vec{E}^d(\vec{H}^d) \tag{18}$$

where the superscripts i, r and d denote the incident, reflected and diffracted fields, respectively. The reflected field components are given by

$$\begin{aligned} E_x^r(x,y,z) &= R_x^e\, E_x^i(x,-y,z) \quad H_x^r(x,y,z) = R_x^m\, H_x^i(x,-y,z) \\ E_z^r(x,y,z) &= R_z^e\, E_z^i(x,-y,z) \quad H_z^r(x,y,z) = R_z^m\, H_z^i(x,-y,z) \end{aligned} \tag{19}$$

where $R_{x,z}^e$ and $R_{x,z}^m$ are the electric and magnetic field reflection coefficients, respectively.
To determine the diffracted fields, electric and magnetic Hertz vectors $\vec{\Pi}^e$ and $\vec{\Pi}^m$ are introduced in the form:

$$\vec{\Pi}^e = (\Pi_x^e, 0, 0) \qquad \text{and} \qquad \vec{\Pi}^m = (\Pi_x^m, 0, 0) \tag{20}$$

with

$$\Pi_x^e = \int_{-\infty}^{\infty} M(\alpha) \exp\{i\alpha z - i\Gamma(\alpha) y\}\, d\alpha \tag{21}$$

$$\Pi_x^m = \int_{-\infty}^{\infty} N(\alpha) \exp\{i\alpha z - i\Gamma(\alpha) y\}\, d\alpha\,. \tag{22}$$

Here $\Gamma(\alpha) = \sqrt{\kappa^2 - \alpha^2}$ with $\kappa^2 = k^2 - k_x^2 = k_y^2 + k_z^2$, and the square-root function is defined in the cut complex α-plane such that $\Gamma = +\kappa$ when $\alpha = 0$ as shown in Fig.-3.
By using the integral expressions (21-22) of the Hertz vectors in the following:

$$\vec{E}^d = \nabla \times \nabla \times \vec{\Pi}^e + ikZ_o \nabla \times \vec{\Pi}^m\,, \tag{23}$$

$$\vec{H}^d = \nabla \times \nabla \times \vec{\Pi}^m - (ik/Z_o) \nabla \times \vec{\Pi}^e\,, \tag{24}$$

the integral representations for the tangential components of the diffracted fields can be written as:

$$\begin{bmatrix} E_x^d(y,z) \\ E_z^d(y,z) \end{bmatrix} = \int_{-\infty}^{\infty} \begin{bmatrix} \kappa^2 & 0 \\ \alpha k_x & -kZ_o\Gamma(\alpha) \end{bmatrix} \begin{bmatrix} M(\alpha) \\ N(\alpha) \end{bmatrix} e^{i\alpha z - i\Gamma(\alpha) y}\, d\alpha \tag{25}$$

and

$$\begin{bmatrix} H_x^d(y,z) \\ H_z^d(y,z) \end{bmatrix} = \int_{-\infty}^{\infty} \begin{bmatrix} 0 & \kappa^2 \\ (k/Z_o)\Gamma(\alpha) & \alpha k_x \end{bmatrix} \begin{bmatrix} M(\alpha) \\ N(\alpha) \end{bmatrix} e^{i\alpha z - i\Gamma(\alpha)y}\, d\alpha. \quad (26)$$

Substitution of the field expressions (25-26) into the boundary conditions, given by (11-12), yields:

$$\int_{-\infty}^{\infty} \left\{ [\kappa^2 - kZ_z^+\Gamma(\alpha)]M(\alpha) + \alpha k_x Z_o Z_z^+ N(\alpha) \right\} e^{i\alpha z} d\alpha =$$

$$-\left\{ A_x^i + A_x^i R_x^e - \frac{Z_o Z_z^+}{\omega\mu}(k_y A_x^i - k_x A_y^i) - \frac{Z_z^+}{\omega\mu} R_z^m (k_y A_x^i - k_x A_y^i) \right\} e^{-ik_z z};\ z > 0, \quad (27)$$

$$\int_{-\infty}^{\infty} \left\{ \alpha k_x M(\alpha) + [Z_o Z\kappa^2 - kZ_o\Gamma(\alpha)]N(\alpha) \right\} e^{i\alpha z}\, d\alpha =$$

$$-\left\{ A_z^i + A_z^i R_z^e - \frac{Z_o Z}{\omega\mu}(k_z A_y^i + k_y A_z^i) - \frac{Z_o Z}{\omega\mu} R_x^m (k_z A_y^i + k_y A_z^i) \right\} e^{-ik_z z};\ z < 0. \quad (28)$$

$$\int_{-\infty}^{\infty} \left\{ [\kappa^2 - kZ_z^-\Gamma(\alpha)]M(\alpha) + \alpha k_x Z_o Z_z^- N(\alpha) \right\} e^{i\alpha z}\, d\alpha =$$

$$-\left\{ A_x^i + A_x^i R_x^e - \frac{Z_o Z_z^-}{\omega\mu}(k_y A_x^i - k_x A_y^i) - \frac{Z_o Z_z^-}{\omega\mu} R_z^m (k_y A_x^i - k_x A_y^i) \right\} e^{-ik_z z};\ z < 0, \quad (29)$$

$$\int_{-\infty}^{\infty} \left\{ \alpha k_x M(\alpha) + [Z_o Z\kappa^2 - kZ_o\Gamma(\alpha)]N(\alpha) \right\} e^{i\alpha z}\, d\alpha =$$

$$-\left\{ A_z^i + A_z^i R_z^e - \frac{Z_o Z}{\omega\mu}(k_z A_y^i + k_y A_z^i) - \frac{Z_o Z}{\omega\mu} R_x^m (k_z A_y^i + k_y A_z^i) \right\} e^{-ik_z z};\ z < 0. \quad (30)$$

The inversion of the above integral equations yields:

$$[\kappa^2 - kZ_z^+\Gamma(\alpha)]M(\alpha) + \alpha k_x Z_o Z_z^+ N(\alpha) = U_1(\alpha) -$$

$$\frac{1}{2\pi}\int_0^{\infty} \left\{ A_x^i + A_x^i R_x^e - \frac{Z_o Z_z^+}{\omega\mu}(k_y A_x^i - k_x A_y^i) - \frac{Z_o Z_z^+}{\omega\mu} R_z^m (k_y A_x^i - k_x A_y^i) \right\} e^{-i(\alpha+k_z)z} dz, \quad (31)$$

$$\alpha k_x M(\alpha) + [Z_o Z\kappa^2 - kZ_o\Gamma(\alpha)]N(\alpha) = U_2(\alpha) -$$

$$\frac{1}{2\pi}\int_0^{\infty} \left\{ A_z^i + A_z^i R_z^e - \frac{Z_o Z}{\omega\mu}(k_z A_y^i + k_y A_z^i) - \frac{Z_o Z}{\omega\mu} R_x^m (k_z A_y^i + k_y A_z^i) \right\} e^{-i(\alpha+k_z)z} dz, \quad (32)$$

$$[\kappa^2 - kZ_z^-\Gamma(\alpha)]M(\alpha) + \alpha k_x Z_o Z_z^- N(\alpha) = L_1(\alpha) -$$

$$\frac{1}{2\pi}\int_{-\infty}^{0}\left\{A_x^i + A_x^i R_x^e - \frac{Z_o Z_z^-}{\omega\mu}(k_y A_x^i - k_x A_y^i) - \frac{Z_o Z_z^-}{\omega\mu}R_z^m(k_y A_x^i - k_x A_y^i)\right\} e^{-i(\alpha+k_z)z}dz, \tag{33}$$

$$\alpha k_x M(\alpha) + [Z_o Z\kappa^2 - kZ_o\Gamma(\alpha)]N(\alpha) = L_2(\alpha) -$$

$$\frac{1}{2\pi}\int_{-\infty}^{0}\left\{A_z^i + A_z^i R_z^e - \frac{Z_o Z}{\omega\mu}(k_z A_y^i + k_y A_z^i) - \frac{Z_o Z}{\omega\mu}R_x^m(k_z A_y^i + k_y A_z^i)\right\} e^{-i(\alpha+k_z)z}dz. \tag{34}$$

$U_{1,2}(\alpha)$ and $L_{1,2}(\alpha)$ are unknown functions which are regular, respectively, in an upper and lower half of the complex α-plane. They correspond to the combinations of the field components determined by the boundary conditions:

$$L_1(\alpha) = \frac{1}{2\pi}\int_0^{\infty}[E_x(x,0,z) - Z_o Z_z^- H_z(x,0,z)]e^{-i\alpha z}\,dz\,, \tag{35}$$

$$L_2(\alpha) = \frac{1}{2\pi}\int_0^{\infty}[E_z(x,0,z) + Z_o Z H_x(x,0,z)]e^{-i\alpha z}\,dz\,, \tag{36}$$

and

$$U_1(\alpha) = \frac{1}{2\pi}\int_{-\infty}^{0}[E_x(x,0,z) - Z_o Z_z^+ H_z(x,0,z)]e^{-i\alpha z}\,dz\,, \tag{37}$$

$$U_2(\alpha) = \frac{1}{2\pi}\int_{-\infty}^{0}[E_z(x,0,z) + Z_o Z H_x(x,0,z)]e^{-i\alpha z}\,dz\,. \tag{38}$$

In order to accomplish the integrations related with the source functions in (31-34), either the integrands of the integrals written for $z \in (0,\infty)$ or for $z \in (-\infty,0)$ should be assumed to be identically equal to zero. Here, the integrands for $z \in (-\infty,0)$ are assumed to vanish:

$$A_x^i(1 + A_x^i R_x^e) - \frac{Z_o Z_z^-}{\omega\mu}(1 + R_z^m)(k_y A_x^i - k_x A_y^i) = 0 \tag{39}$$

$$A_z^i(1 + A_z^i R_z^e) - \frac{Z_o Z}{\omega\mu}(1 + R_x^m)(k_z A_y^i + k_y A_z^i) = 0 \tag{40}$$

This implies that the reflected field terms inserted in the total field representation correspond to the fields that would be reflected from a full impedance plane with the surface impedance values being imposed as on the $z < 0$ part of the original configuration. During the inversion of the integrals in the interval $z \in (0,\infty)$, the integrations are accomplished in a strip of regularity D, where $D = D_U \cap D_L$ with $D_L = \{\alpha \mid Im(\alpha) < Im(-k_z)\}$ and $D_U = \{\alpha \mid Im(\alpha) > Im(-\kappa)\}$ as shown in Fig.-3.

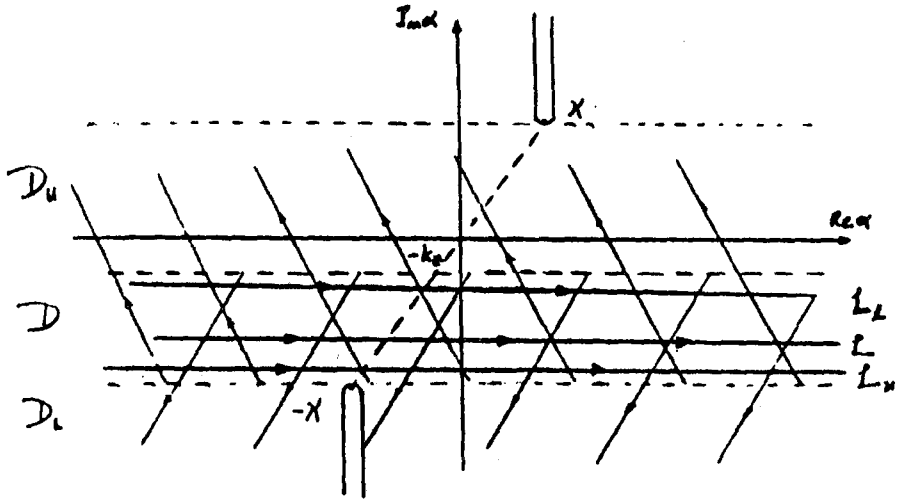


Figure-3. The regularity band in the complex α-plane.

Therefore, by expressing (31-34) in matrix form

$$\begin{bmatrix} \kappa^2 - kZ_z^+\Gamma(\alpha) & -\alpha k_x Z_o Z_z^+ \\ \alpha k_x & Z_o Z\kappa^2 - kZ_o\Gamma(\alpha) \end{bmatrix} \begin{bmatrix} M(\alpha) \\ N(\alpha) \end{bmatrix} = \begin{bmatrix} U_1(\alpha) \\ U_2(\alpha) \end{bmatrix} + \begin{bmatrix} V_1(\alpha) \\ V_2(\alpha) \end{bmatrix} \quad (41)$$

$$\begin{bmatrix} \kappa^2 - kZ_z^-\Gamma(\alpha) & -\alpha k_x Z_o Z_z^- \\ \alpha k_x & Z_o Z\kappa^2 - kZ_o\Gamma(\alpha) \end{bmatrix} \begin{bmatrix} M(\alpha) \\ N(\alpha) \end{bmatrix} = \begin{bmatrix} L_1(\alpha) \\ L_2(\alpha) \end{bmatrix}. \quad (42)$$

is obtained. Eliminating the unknown coefficients $M(\alpha)$ and $N(\alpha)$, two coupled functional equations of the Wiener-Hopf type are obtained. They can be expressed in matrix form as follows:

$$\mathbf{G}(\alpha)\mathbf{L}(\alpha) = \mathbf{U}(\alpha) + \mathbf{V}(\alpha) \quad (43)$$

where

$$\mathbf{G}(\alpha) = \frac{1}{\Delta(\alpha)} \begin{bmatrix} Z\kappa^2 + kb\Gamma(\alpha) + Z_z^+(k^2 - \alpha^2) & \alpha k_x(Z_z^- - Z_z^+) \\ 0 & Z\kappa^2 + ka\Gamma(\alpha) + Z_z^-(k^2 - \alpha^2) \end{bmatrix} \quad (44)$$

with

$$a = -(1 + ZZ_z^-) \quad , \qquad b = -(1 + ZZ_z^+) \tag{45}$$

and

$$\Delta(\alpha) = \{Z\kappa^2 + ka\Gamma(\alpha) + Z_z^-(k^2 - \alpha^2)\}. \tag{46}$$

$\mathbf{U}(\alpha), \mathbf{L}(\alpha)$ and $\mathbf{V}(\alpha)$ are column vectors, where the terms $\mathbf{U}(\alpha), \mathbf{L}(\alpha)$ are given by (35-36) and (37-38), respectively, and $\mathbf{V}(\alpha)$ corresponds to the contributions of the incident and reflected fields:

$$V_1(\alpha) = -\frac{1}{2\pi i(\alpha + k_z)}\{A_x^i(1 + A_x^i R_x^e) - \frac{Z_o Z_z^+}{\omega\mu}(1 + R_z^m)(k_y A_x^i - k_x A_y^i)\}, \tag{47}$$

$$V_2(\alpha) = -\frac{1}{2\pi i(\alpha + k_z)}\{A_z^i(1 + A_z^i R_z^e) - \frac{Z_o Z}{\omega\mu}(1 + R_x^m)(k_z A_y^i + k_y A_z^i)\}. \tag{48}$$

4. FACTORIZATION OF THE KERNEL MATRIX

As is known, the solution of a matrix Wiener-Hopf system is identical to the solution procedure of a scalar Wiener-Hopf equation. The classical Wiener-Hopf procedure requires the factorization of the kernel matrix $\mathbf{G}(\alpha)$ in the following form:

$$\mathbf{G}(\alpha) = \mathbf{G}_L(\alpha)\,\mathbf{G}_U(\alpha) \tag{49}$$

where $\mathbf{G}_U(\alpha)$ and $\mathbf{G}_L(\alpha)$ and their inverses are regular functions of α with algebraic behaviour for $\alpha \to \infty$ in the half-planes $\mathcal{D}_U$ and $\mathcal{D}_L$, respectively. When the kernel function is a scalar, a general factorization formula is obtained by taking the logarithm of the function and writing it as the sum of the functions analytic in $\mathcal{D}_U$ and $\mathcal{D}_L$ with the aid of Cauchy's theorem. Then the appropriate factors can easily be obtained by taking exponentials. However, this method makes use of the commutativity of the scalar functions, and of course, cannot be extended to matrices which in general do not form a commutative algebra. This is the main source of difficulty in solving matrix Wiener-Hopf equations.

Although the problem of the Wiener-Hopf factorization of an arbitrary matrix still remains at present an open problem, a significant amount of progress has been achieved in the last three decades for the solution of a certain type of matrix Wiener-Hopf equations. One of the relatively general procedures is the direct factorization method which is applicable even to those cases where the kernel matrix has only poles or poles and branch point singularities. Chebotarev [10], Khrapkov [11] and Daniele [12] have contributed to the development of this method and it is known as Daniele-Khrapkov method of factorization. First, the fundamentals of this method will be given in this section and then it will be applied to the problem

under consideration.

4.1. Fundamentals of the Daniele-Khrapkov Method of Factorization

This method relies on the logarithmic approach, and in general, derives the multiplicative factors of a scalar function from the additive splitting of its logarithm. When this concept is applied to a matrix

$$\mathbf{G}(\alpha) = \exp\{\ln \mathbf{G}(\alpha)\} \tag{50}$$

is written, where

$$\ln \mathbf{G}(\alpha) = \{\ln \mathbf{G}(\alpha)\}_U + \{\ln \mathbf{G}(\alpha)\}_L \tag{51}$$

then

$$\mathbf{G}(\alpha) = \mathbf{G}_L(\alpha)\mathbf{G}_U(\alpha) = \mathbf{G}_U(\alpha)\mathbf{G}_L(\alpha) \tag{52}$$

is obtained, with

$$\mathbf{G}_U(\alpha) = \exp\{\ln \mathbf{G}(\alpha)\}_U, \text{ and } \mathbf{G}_L(\alpha) = \exp\{\ln \mathbf{G}(\alpha)\}_L. \tag{53}$$

However, it is obvious that (52-53) necessitates the commutativity of the factor matrices which may not always be the case.
Here, the Daniele-Khrapkov method will be explained by considering the class of matrices given in the form

$$\mathbf{G}(\alpha) = a_1(\alpha)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + a_2(\alpha)\begin{bmatrix} l_1(\alpha) & m(\alpha) \\ n(\alpha) & l_2(\alpha) \end{bmatrix}. \tag{54}$$

where $a_{1,2}(\alpha)$ are scalar functions and $l_{1,2}(\alpha)$, $m(\alpha)$ and $n(\alpha)$ are polynomials. First step is to calculate the trace of the matrix $\mathbf{G}(\alpha)$, which is equal to the sum of the diagonal elements; then half of the trace will be added and subtracted from $\mathbf{G}(\alpha)$ in order to obtain a more convenient form, as:

$$\mathbf{G}(\alpha) = s(\alpha)\mathbf{I} + a_2(\alpha)\mathbf{J}(\alpha). \tag{55}$$

Here, $s(\alpha)$ denotes the half of the trace of $\mathbf{G}(\alpha)$ and is obtained as

$$s = a_1(\alpha) + \frac{1}{2}a_2(\alpha)[l_1(\alpha) + l_2(\alpha)]$$

and $\mathbf{J}(\alpha)$ is a polynomial matrix as

$$\mathbf{J}(\alpha) = \begin{bmatrix} l(\alpha) & m(\alpha) \\ n(\alpha) & -l(\alpha) \end{bmatrix} \tag{56}$$

with $l(\alpha) = (1/2)[l_1(\alpha) - l_2(\alpha)]$.
For the subsequent factorization procedure it is convenient to express the matrix $\mathbf{G}(\alpha)$ in the following form:

$$\mathbf{G}(\alpha) = \sqrt{W(\alpha)}[\cosh(\sqrt{f}F)\mathbf{I} + (1/g\sqrt{f})\sinh(\sqrt{f}F)\mathbf{J}(\alpha)] \tag{57}$$

where $g = 1$ for the present case and $W(\alpha)$ denotes the determinant in terms of the eigenvalues $\lambda_{1,2}(\alpha)$ of the matrix $\mathbf{G}(\alpha)$ as

$$W(\alpha) = \lambda_1(\alpha)\lambda_2(\alpha)$$

with

$$\lambda_{1,2}(\alpha) = a_1(\alpha) \pm a_2(\alpha)\sqrt{f(\alpha)}.$$

The definition of $F(\alpha)$ is

$$F(\alpha) = \frac{1}{2}.\frac{1}{\sqrt{f(\alpha)}}\ln[\frac{\lambda_1(\alpha)}{\lambda_2(\alpha)}], \tag{58}$$

where

$$f(\alpha) = l^2(\alpha) + m(\alpha)n(\alpha).$$

Now, the explicit expresssions of the split matrices can easily be written as

$$\mathbf{G}_{U,L}(\alpha) = \sqrt{W_{U,L}(\alpha)}[\cosh(\sqrt{f}F_{U,L})\mathbf{I} + (1/\sqrt{f})\sinh(\sqrt{f}F_{U,L})\mathbf{J}(\alpha)]. \tag{59}$$

Here $W_{U,L}(\alpha)$ and $F_{U,L}(\alpha)$ are functions resulting from the multiplicative and additive split of $W(\alpha)$ and $F(\alpha)$, respectively, i.e.,

$$W(\alpha) = W_U(\alpha)W_L(\alpha), \tag{60}$$

and

$$F(\alpha) = F_U(\alpha) + F_L(\alpha). \tag{61}$$

A difficulty which sometimes arises in the application of this method is that the split matrices may be exponentially large in their respective regions of regularity. By denoting the order of the polynomial $f(\alpha)$ with q, it is supposed that

$$f(\alpha) = O(\alpha^q) \qquad \text{as} \qquad |\,\alpha\,| \rightarrow \infty. \tag{62}$$

It can easily be seen that for $q > 2$, the factor matrices display an exponential behaviour.

Daniele [13] showed that the exponentially growing terms appearing in the split matrices can be cancelled by finding a polynomial matrix of the form

$$\mathbf{P}(\alpha) = p_1(\alpha)\mathbf{I} + p_2(\alpha)\mathbf{J}(\alpha) \tag{63}$$

where p_1, p_2 are polynomials of α with undetermined coefficients. Then to cancel the exponentially growing terms, $\mathbf{G}(\alpha)$ will be multiplied by the polynomial matrix and its inverse in the following form

$$\mathbf{G}(\alpha) = \mathbf{G}(\alpha)\mathbf{P}^{-1}(\alpha)\mathbf{P}(\alpha) = \mathbf{G}_P(\alpha)\mathbf{P}(\alpha). \tag{64}$$

It is also assumed that the multiplicative factors of $\mathbf{P}(\alpha)$ have the same exponential behaviour as the factors of $\mathbf{G}(\alpha)$. Since $\mathbf{G}(\alpha), \mathbf{P}(\alpha)$ and their inverses and multiplicative factors all have the same commutant, they commute, so

$$\mathbf{G}(\alpha) = \mathbf{G}_{PL}(\alpha)\mathbf{P}(\alpha)\mathbf{G}_{PU}(\alpha) \tag{65}$$

can be written. Thus, the factors are as follows:

$$\mathbf{G}_{PU}(\alpha) = \mathbf{P}_U^{-1}(\alpha)\mathbf{G}_U(\alpha) = \sqrt{\frac{W_U}{(\det\mathbf{P})_\mathbf{U}}} \times$$
$$\{\cosh[\sqrt{f}(F_U - F_{PU})]\mathbf{I} + \sqrt{f}\sinh[\sqrt{f}(F_U - F_{PU})]\mathbf{J}(\alpha)\}, \tag{66}$$
$$\mathbf{G}_{PL}(\alpha) = \mathbf{G}_L(\alpha)\mathbf{P}_L^{-1}(\alpha) = \sqrt{\frac{W_L}{(\det\mathbf{P})_L}} \times$$
$$\{\cosh[\sqrt{f}(F_L - F_{PL})]\mathbf{I} + \sqrt{f}\sinh[\sqrt{f}(F_L - F_{PL})]\mathbf{J}(\alpha)\}. \tag{67}$$

Here, F_{PU} and F_{PL} denote the additive split factors of

$$F_P(\alpha) = \frac{1}{2}.\frac{1}{\sqrt{f(\alpha)}}\ln(\frac{\lambda_{p1}}{\lambda_{p2}})$$

which corresponds to the polynomial matrix given in (63), with

$$\lambda_{p1,p2} = p_1(\alpha) \pm p_2(\alpha)\sqrt{f(\alpha)}$$

showing the eigenvalues.

4.2. Factorization by Daniele-Khrapkov Method

Now, the factorization of the kernel matrix given by (44) will be accomplished by Daniele-Khrapkov method. Since one of the elements of $\mathbf{G}(\alpha)$ is zero, it should

be noted that matrix Wiener-Hopf system given by (43) can be solved by successive elimination yielding to a scalar Wiener-Hopf equation (see Appendix).
Let the matrix $\mathbf{G}(\alpha)$ be written as

$$\mathbf{G}(\alpha) = \frac{k}{\Delta(\alpha)}\Gamma(\alpha)\mathbf{B}\hat{\mathbf{G}}(\alpha) \tag{68}$$

with $\mathbf{B}$ being a constant matrix

$$\mathbf{B} = \begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix}$$

and

$$\hat{\mathbf{G}}(\alpha) = \mathbf{I} + a_2(\alpha)\begin{bmatrix} \ell_1(\alpha) & m(\alpha) \\ 0 & \ell_2(\alpha) \end{bmatrix}$$

where $\mathbf{I}$ is the unit matrix and

$$\begin{aligned} a_2(\alpha) &= [ka\,b\Gamma(\alpha)]^{-1}, & m(\alpha) &= a(Z_z^- - Z_z^+)\,\alpha k_x \\ \ell_1(\alpha) &= aZ\kappa^2 + aZ_z^+(k^2-\alpha^2), & \ell_2(\alpha) &= bZ\,\kappa^2 + bZ_z^-(k^2-\alpha^2). \end{aligned} \tag{69}$$

Since $\mathbf{B}$ is a scalar matrix, only the factorization of $\hat{\mathbf{G}}(\alpha)$ will be considered. By adding and subtracting $s(\alpha)$, it will be written as follows in Daniele-Khrapkov form:

$$\hat{\mathbf{G}}(\alpha) = s(\alpha)\mathbf{I} + a_2(\alpha)\mathbf{J}(\alpha) \tag{70}$$

where

$$\mathbf{J}(\alpha) = \begin{bmatrix} \ell(\alpha) & m(\alpha) \\ 0 & -\ell(\alpha) \end{bmatrix}$$

with

$$s(\alpha) = [2abk\Gamma(\alpha)]^{-1}\{2abk\Gamma(\alpha) + (aZ_z^+ + bZ_z^-)(k^2-\alpha^2) + (a+b)Z\,\kappa^2\} \tag{71}$$

and

$$\ell(\alpha) = (1/2)\{(aZ_z^+ - bZ_z^-)(k^2-\alpha^2) + Z(a-b)\kappa^2\}. \tag{72}$$

The eigenvalues of the matrix $\hat{\mathbf{G}}(\alpha)$ are:

$$\lambda_1(\alpha) = [ak\Gamma(\alpha)]^{-1}\{ak\Gamma(\alpha) + Z\kappa^2 + Z_z^-(k^2-\alpha^2)\} \tag{73}$$

$$\lambda_2(\alpha) = [bk\Gamma(\alpha)]^{-1}\{bk\Gamma(\alpha) + Z\kappa^2 + Z_z^+(k^2-\alpha^2)\}. \tag{74}$$

As known the factorization of the kernel matrix requires the multiplicative split of the determinat

$$W(\alpha) = W_U(\alpha)W_L(\alpha)\,, \tag{75}$$

and the additive split of

$$F(\alpha) = F_U(\alpha) + F_L(\alpha)\,. \tag{76}$$

Now, in order to perform the multiplicative split of the determinant:

$$W(\alpha) = \lambda_1(\alpha)\lambda_2(\alpha), \tag{77}$$

it is helpful to introduce the following equality related with $\lambda_1(\alpha)$:

$$-Z_z^-\alpha^2 + ak\Gamma(\alpha) + Z\kappa^2 + Z_z^- k^2 = \frac{Z_z^-}{\xi_1\xi_2}\frac{\Gamma(\alpha)}{\chi(\xi_1,\alpha)\,\chi(\xi_2,\alpha)} \tag{78}$$

with

$$\xi_1 = \kappa/\sigma_1 \quad \text{and} \quad \xi_2 = \kappa/\sigma_2, \tag{79}$$

where

$$\sigma_{1,2} = \frac{-\gamma_1}{2} \pm \frac{1}{2}\sqrt{{\gamma_1}^2 - 4\gamma_2},$$

and

$$\gamma_1 = \frac{-ak}{Z_z^-}, \quad \gamma_2 = -\kappa^2 + k^2 + \frac{Z}{Z_z^-}\kappa^2.$$

Naturally, a similar expression is written for $\lambda_2(\alpha)$, with the following :

$$\xi_3 = \kappa/\sigma_3 \quad ; \qquad \xi_4 = \kappa/\sigma_4, \tag{80}$$

$$\sigma_{3,4} = \frac{-\gamma_3}{2} \pm \frac{1}{2}\sqrt{{\gamma_3}^2 - 4\gamma_4},$$

and

$$\gamma_3 = \frac{-bk}{Z_z^+}, \quad \gamma_4 = -\kappa^2 + k^2 + \frac{Z}{Z_z^+}\kappa^2.$$

The function $\chi(\xi,\alpha)$ given in (78) is defined as

$$\chi(\xi,\alpha) = \frac{\Gamma(\alpha)}{\kappa + \xi\Gamma(\alpha)} = \chi_U(\xi,\alpha)\chi_L(\xi,\alpha) \tag{81}$$

which is factorized in terms of Maliuzhinetz function (see [14]). So, the factors of $W(\alpha)$ read

$$W_{U,L}(\alpha) = \frac{1}{k\sqrt{ab}}\cdot\frac{\sqrt{Z_z^- Z_z^+}}{\sqrt{\xi_1\,\xi_2\,\xi_3\,\xi_4}}\cdot\frac{\kappa \pm \alpha}{\chi_{U,L}(\xi_1,\alpha)\,\chi_{U,L}(\xi_2,\alpha)\,\chi_{U,L}(\xi_3,\alpha)\,\chi_{U,L}(\xi_4,\alpha)}\,. \tag{82}$$

For the additive split of $\mathbf{F}(\alpha)$, the usual integral definition will be used,

$$F_{U,L}(\alpha) = \pm\frac{1}{2\pi i}\int_{\mathcal{L}^{\pm}} \frac{1}{2\sqrt{\ell(t)}} \ln\frac{\lambda_1(t)}{\lambda_2(t)} \cdot \frac{dt}{(t-\alpha)}, \tag{83}$$

where the integration lines are taken as shown in Fig.-3. By considering that the integrand does not have any poles, the integrations can be performed easily to give

$$F_{U,L}(\alpha) = \frac{1}{2\ell(\alpha)} \ln\frac{\lambda_{1U,L}(\alpha)}{\lambda_{2U,L}(\alpha)}. \tag{84}$$

Here $\lambda_{jU}(\alpha)$ and $\lambda_{jL}(\alpha)$, $j = 1, 2$, denote the multiplicative factors of the eigenvalues which can easily be obtained from (77-78).
Now, the factors of $\hat{\mathbf{G}}(\alpha)$ can be written as

$$\hat{\mathbf{G}}_{U,L}(\alpha) = \sqrt{W_{U,L}(\alpha)}\left\{\cosh[\sqrt{f}\,F_{U,L}(\alpha)]\mathbf{I} + \frac{1}{\sqrt{f}}\sinh[\sqrt{f}\,F_{U,L}(\alpha)]\mathbf{J}(\alpha)\right\}. \tag{85}$$

Since $f(\alpha)$ is a fourth order polynomial, it is obvious that the factor matrices will have an exponential behaviour. The next step is to introduce a polynomial matrix in the form given by (63) to cancel this exponential growth.
As known , the order of this polynomial matrix will be at the order of $\sqrt{f}$ and since it is a second order polynomial for the problem under consideration, it will involve only one unknown coefficient to be determined with the aid of the regularity condition. Regarding these points $\mathbf{P}(\alpha)$ can be written as

$$\mathbf{P}(\alpha) = (\delta - \alpha^2)\mathbf{I} + \mathbf{J}(\alpha), \tag{86}$$

and the eigenvalues of the matrix $\mathbf{P}(\alpha)$ read

$$\lambda_{p1,p2}(\alpha) = (\delta - \alpha^2) \pm \ell(\alpha). \tag{87}$$

Now, δ is the scalar constant which is to be determined by the regularity condition given as:

$$\int_{-\infty}^{\infty}\{F(t) - F_p(t)\}dt = 0. \tag{88}$$

This yields

$$\int_{-\infty}^{\infty}\frac{1}{2}\cdot\frac{1}{\sqrt{f(t)}}\left\{\ln[\frac{\lambda_1(t)}{\lambda_2(t)}] - \ln[\frac{\lambda_{P1}(t)}{\lambda_{P2}(t)}]\right\}dt = 0 \tag{89}$$

and by substituting the expressions for the eigenvalues

$$\int_{-\infty}^{\infty}\ln\left\{\frac{b}{a}\cdot[\frac{ak\Gamma(t) + Z\kappa^2 + Z_z^-(k^2-t^2)}{bk\Gamma(t) + Z\kappa^2 + Z_z^+(k^2-t^2)}]\right\}\frac{dt}{l(t)} =$$

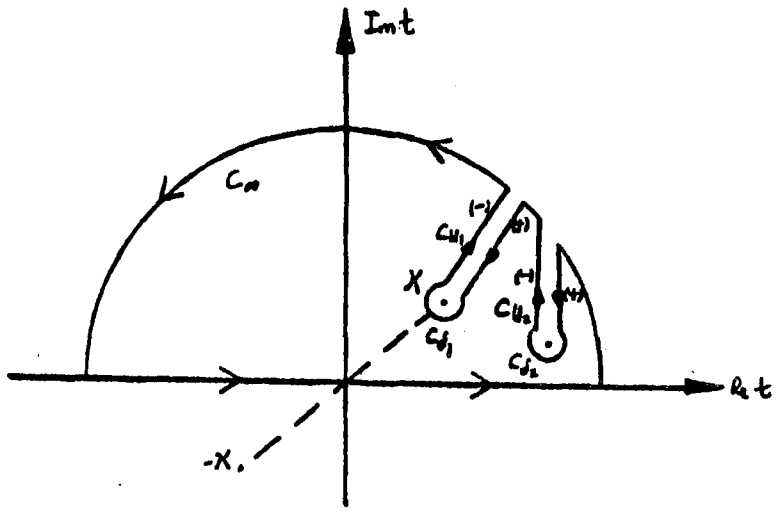


Figure-4. Branch points and the integration contours for evaluation of the integrals $\mathcal{I}_A$ and $\mathcal{I}_B$.

$$\int_{-\infty}^{\infty} \ln\left\{\frac{\delta - t^2 + 1/2[(aZ_z^+ - bZ_z^-)(k^2 - t^2) + Z(a-b)\kappa^2]}{\delta - t^2 - 1/2[(aZ_z^+ - bZ_z^-)(k^2 - t^2) + Z(a-b)\kappa^2]}\right\} \frac{dt}{l(t)}. \qquad (90)$$

is obtained.

Let the integrals on the left hand side and the right hand side of (90) be denoted by $\mathcal{I}_A$ and $\mathcal{I}_B$, respectively. The zeros of the polynomial $l(t)$ can easily be calculated as

$$t_o = \pm\left\{k^2 + \frac{(a-b)Z\kappa^2}{(aZ_z^+ - bZ_z^-)}\right\}^{1/2}.$$

For the integral $\mathcal{I}_A$, t_o is not either a pole or branch-point because $\lambda_1(t_o) = \lambda_2(t_o)$. So, only singularity comes from $\Gamma(t)$ yielding $t = \pm\kappa$ as branch points. The path of integration for $\mathcal{I}_A$ will be deformed into a contour $C_{U\infty}$ enclosing the branch point $t = \kappa$ as shown in Fig.-4. It can easily be shown that the integrals along C_∞ and C_{δ_∞} tend to zero when the radii of these circles tend to infinity and zero, respectively. Then, let the two sides of the branch cuts be denoted by (+) and (-)

supercripts, so it is possible to obtain

$$\mathcal{I}_A = \int_{C^+_{U_1}} \ln\left\{\frac{\lambda_1(t)}{\lambda_2(t)}\sqrt{\kappa+t} \mid \sqrt{\kappa+t} \mid e^{i\pi/4}\right\}\frac{dt}{l(t)} +$$

$$\int_{C^-_{U_1}} \ln\left\{\frac{\lambda_1(t)}{\lambda_2(t)}\sqrt{\kappa+t} \mid \sqrt{\kappa+t} \mid e^{-i3\pi/4}\right\}\frac{dt}{l(t)} \tag{91}$$

or

$$\mathcal{I}_A = \int_k^{i\infty} \ln\left\{\frac{\lambda_1(t)}{\lambda_2(t)}\sqrt{\kappa+t} \mid \sqrt{\kappa+t} \mid e^{i\pi/4}\right\}\frac{dt}{l(t)} -$$

$$\int_k^{i\infty} \ln\left\{\frac{\lambda_1(t)}{\lambda_2(t)}\sqrt{\kappa+t} \mid \sqrt{\kappa+t} \mid e^{-i3\pi/4}\right\}\frac{dt}{l(t)} - \int_k^{i\infty} \ln[e^{-i\pi}]\frac{dt}{l(t)} \tag{92}$$

and this gives

$$\mathcal{I}_A = 2i\pi \int_k^{i\infty} \frac{dt}{(bZ_z^- - aZ_z^+)(k^2-t^2) + Z(b-a)\kappa^2}$$

evaluation of integral

$$\mathcal{I}_A = \frac{-2\pi i}{\nu_1}\arctan(k/\nu_2) \tag{93}$$

where

$$\nu_1 = (aZ_z^+ - bZ_z^-) \quad \nu_2 = \sqrt{\frac{Z(b-a)\kappa^2}{\nu_1} - k^2}.$$

In order to calculate the integral $\mathcal{I}_B$, the integration line will be deformed to a contour C_{U_2} enclosing the branch point $t = t_o$. It can again easily be shown that the contributions from the contours C_∞ and $C_{\delta 2}$ are equal to zero. Therefore,

$$\mathcal{I}_B = \int_{C^+_{U_2}} \ln\left\{\frac{\delta - t^2 + l(t)}{\delta - t^2 - l(t)}e^{i\pi/2}\right\}\frac{dt}{l(t)} -$$

$$\int_{C^-_{U_2}} \ln\left\{\frac{\delta - t^2 + l(t)}{\delta - t^2 - l(t)}e^{-i3\pi/2}\right\}\frac{dt}{l(t)}, \tag{94}$$

where (+) and (−) superscripts again denote the two sides of the branch cut. Cancelling similar terms in the above integrals yields

$$\mathcal{I}_B = \frac{-i\pi}{\nu_1}\arctan(k/\nu_2) \tag{95}$$

So, the constants t_o and δ are determined as follows:

$$t_o = \nu_2 \tan[\frac{1}{2}\arctan(k/\nu_2)] \tag{96}$$

$$\delta = t_o^2[1 + 1/2(bZ_z^- - aZ_z^+)] - 1/2(bZ_z^- - aZ_z^+)k^2 - (1/2)Z(b-a)\kappa^2. \quad (97)$$

The additive and multiplicative splits related with the polynomial matrix can easily be performed. So, the factors of the matrix product in (66) are

$$(\hat{\mathbf{G}}(\alpha)\mathbf{P}^{-1})_{U,L} = \sqrt{\frac{\lambda_{1U,L}\,\lambda_{2U,L}}{\lambda_{p1U,L}\,\lambda_{p2U,L}}}$$

$$\left\{\mathbf{I}\cosh[\sqrt{f}(F_{U,L} - F_{pU,L})] + \mathbf{C}\sinh[\sqrt{f}(F_{U,L} - F_{pU,L})]\right\} \quad (98)$$

Now, the matrix given by (64) can be written as

$$\hat{\mathbf{G}}(\alpha)\mathbf{P}^{-1}(\alpha)\mathbf{P}(\alpha) = [\hat{\mathbf{G}}(\alpha)\mathbf{P}^{-1}(\alpha)]_L\,\mathbf{P}(\alpha)\,[\hat{\mathbf{G}}(\alpha)\mathbf{P}^{-1}(\alpha)]_U, \quad (99)$$

and it remains to factorize the polynomial matrix:

$$\mathbf{P}(\alpha) = \begin{bmatrix} \delta - \alpha^2 + \ell(\alpha) & m(\alpha) \\ 0 & \delta - \alpha^2 - \ell(\alpha) \end{bmatrix}. \quad (100)$$

By inspection,

$$\mathbf{P}(\alpha) = \mathbf{P}_L(\alpha)\,\mathbf{P}_U(\alpha) \quad (101)$$

is obtained with

$$\mathbf{P}_U(\alpha) = \begin{bmatrix} \sqrt{1-l_1}(c_1 - \alpha) & d_1 \\ 0 & \sqrt{1+l_1}(c_2 - \alpha) \end{bmatrix} \quad (102)$$

and

$$\mathbf{P}_L(\alpha) = \begin{bmatrix} \sqrt{1-l_1}(c_1 + \alpha) & d_2 \\ 0 & \sqrt{1+l_1}(c_2 + \alpha) \end{bmatrix}. \quad (103)$$

Here

$$d_1 = \frac{c_1 b k_x (Z_z^+ - Z_z^-)}{\sqrt{1+l_1}(c_1 + c_2)}, \quad d_2 = \frac{-c_2 b k_x (Z_z^+ - Z_z^-)}{\sqrt{1-l_1}(c_1 + c_2)},$$

and

$$c_1 = \sqrt{(\delta + l_o)/(1 - l_1)}, \quad c_2 = \sqrt{(\delta - l_o)/(1 + l_1)},$$

are written. The function $\ell(\alpha)$ is expressed as

$$\ell = l_o + \alpha^2 l_1,$$

with

$$l_o = \frac{1}{2}[Z\kappa^2(b-a) + k^2(bZ_Z^- - aZ_z^+)] \quad l_1 = \frac{1}{2}(aZ_Z^+ - bZ_z^-).$$

Thus, (44) is written as:

$$\mathbf{G}(\alpha) = \frac{k}{\Delta(\alpha)}\Gamma(\alpha)\mathbf{B}[\hat{\mathbf{G}}\mathbf{P}^{-1}(\alpha)]_L\,\mathbf{P}_L(\alpha)\,\mathbf{P}_U(\alpha)\,[\hat{\mathbf{G}}\mathbf{P}^{-1}(\alpha)]_U \tag{104}$$

with

$$\mathbf{G}_L(\alpha) = \left\{\frac{\sqrt{k}\sqrt{\kappa-\alpha}}{\Delta_L(\alpha)}\mathbf{B}[\hat{\mathbf{G}}(\alpha)\mathbf{P}^{-1}(\alpha)]_L\,\mathbf{P}_L(\alpha)\right\} \tag{105}$$

$$\mathbf{G}_U(\alpha) = \left\{\frac{\sqrt{k}\sqrt{\kappa+\alpha}}{\Delta_U(\alpha)}\mathbf{P}_U(\alpha)[\hat{\mathbf{G}}(\alpha)\mathbf{P}^{-1}(\alpha)]_U\right\}. \tag{106}$$

The expressions of the factor matrices are:

$$\mathbf{G}_L(\alpha) = \frac{\sqrt{k}\sqrt{\kappa-\alpha}}{\Delta_L(\alpha)}\sqrt{\frac{\lambda_{1L}\lambda_{2L}}{\lambda_{p1L}\lambda_{p2L}}}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$\times\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cosh\left[\frac{1}{2}\ln\frac{\lambda_{1L}\lambda_{p2L}}{\lambda_{2L}\lambda_{p1L}}\right] + \frac{1}{l}\begin{bmatrix} l & m \\ 0 & -l \end{bmatrix}\sinh\left[\frac{1}{2}\ln\frac{\lambda_{1L}\lambda_{p2L}}{\lambda_{2L}\lambda_{p1L}}\right]\right\}\mathbf{P}_L(\alpha) \tag{107}$$

and

$$\mathbf{G}_U(\alpha) = \frac{\sqrt{k}\sqrt{\kappa+\alpha}}{\Delta_U(\alpha)}\sqrt{\frac{\lambda_{1U}\lambda_{2U}}{\lambda_{p1U}\lambda_{p2U}}}\mathbf{P}_U(\alpha)\times$$

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cosh\left[\frac{1}{2}\ln\frac{\lambda_{1U}\lambda_{p2U}}{\lambda_{2U}\lambda_{p1U}}\right] + \frac{1}{l}\begin{bmatrix} l & m \\ 0 & -l \end{bmatrix}\sinh\left[\frac{1}{2}\ln\frac{\lambda_{1U}\lambda_{p2U}}{\lambda_{2U}\lambda_{p1U}}\right]\right\}. \tag{108}$$

5. SOLUTION OF THE MATRIX WIENER-HOPF SYSTEM

By substituting (49) into (43)

$$\mathbf{G}_L(\alpha)\,\mathbf{L}(\alpha) = [\mathbf{G}_U(\alpha)]^{-1}\,\mathbf{U}(\alpha) + [\mathbf{G}_U(\alpha)]^{-1}\,\mathbf{V}(\alpha) \tag{109}$$

is obtained. by defining the following matrix as the contibution of the source terms

$$\mathbf{W}(\alpha) = [\mathbf{G}_U(\alpha)]^{-1}\mathbf{V}(\alpha) = (\alpha + k_z)^{-1}[\mathbf{G}_L(\alpha)]^{-1}\mathbf{V}_\mathbf{o},$$

additive decomposition is necessary

$$\mathbf{W}(\alpha) = \mathbf{W}_U(\alpha) + \mathbf{W}_L(\alpha) \tag{110}$$

according to the standard procedure.

Here, $\mathbf{W}_U(\alpha)$ and $\mathbf{W}_L(\alpha)$ are defined as follows:

$$\mathbf{W}_U(\alpha) = \frac{1}{2\pi i}\int_{\mathcal{L}_U}(t+k_z)^{-1}\mathbf{G}_U^{-1}(t)\mathbf{V}_o\frac{dt}{(t-\alpha)}, \tag{111}$$

$$\mathbf{W}_L(\alpha) = -\frac{1}{2\pi i}\int_{\mathcal{L}_\mathcal{L}}(t+k_z)^{-1}\mathbf{G}_U^{-1}(t)\mathbf{V}_o\frac{dt}{(t-\alpha)}. \tag{112}$$

Considering the complex α-plane and the integration lines defined in the plane as shown in Fig.-3.

$$\mathbf{W}_L(\alpha) = (\alpha+k_z)^{-1}\mathbf{G}_U^{-1}(-k_z)\mathbf{V}_o \tag{113}$$

and

$$\mathbf{W}_U(\alpha) = -(\alpha+k_z)^{-1}\{\mathbf{G}_U^{-1}(\alpha) - \mathbf{G}_U^{-1}(-k_z)\}\mathbf{V}_o \tag{114}$$

are obtained. Then substituting these expressions into (109)

$$\mathbf{U}(\alpha) = (\alpha+k_z)^{-1}\left[\mathbf{G}_U^{-1}(-k_z)\mathbf{G}_U^{-1}(\alpha) - \mathbf{I}\right]\mathbf{V}_o, \tag{115}$$

and

$$\mathbf{L}(\alpha) = (\alpha+k_z)^{-1}\mathbf{G}_U^{-1}(-k_z)\mathbf{G}_L^{-1}(\alpha)\mathbf{V}_o, \tag{116}$$

are written with

$$\mathbf{V}_o = \frac{-1}{2\pi i}\begin{bmatrix} A_x^i + A_x^i R_x^e - \dfrac{Z_o Z_z^+}{\omega\mu}(1+R_z^m)(k_y A_x^i - k_x A_y^i) \\ A_z^i + A_z^i R_z^e - \dfrac{Z_o Z}{\omega\mu}(1+R_x^m)(k_z A_y^i + k_y A_z^i) \end{bmatrix}. \tag{117}$$

Now, going back to the equations that are obtained by the application of the integral representations of the fields to the boundary conditions, the unknown spectral functions can easily be expressed in terms of the upper or lower analytic functions which are determined by the solution of the Wiener-Hopf system. Simple calculations yield both of the spectral coefficients in terms of $L_1(\alpha)$ and $L_2(\alpha)$ as follows:

$$\begin{bmatrix} M(\alpha) \\ N(\alpha) \end{bmatrix} = \Delta^{-1}(\alpha)\begin{bmatrix} Z_o Z\kappa^2 - kZ_o\Gamma(\alpha) & \alpha k_x Z_o Z_z^- \\ -\alpha k_x & \kappa^2 - kZ_z^-\Gamma(\alpha) \end{bmatrix}\begin{bmatrix} L_1(\alpha) \\ L_2(\alpha) \end{bmatrix} \tag{118}$$

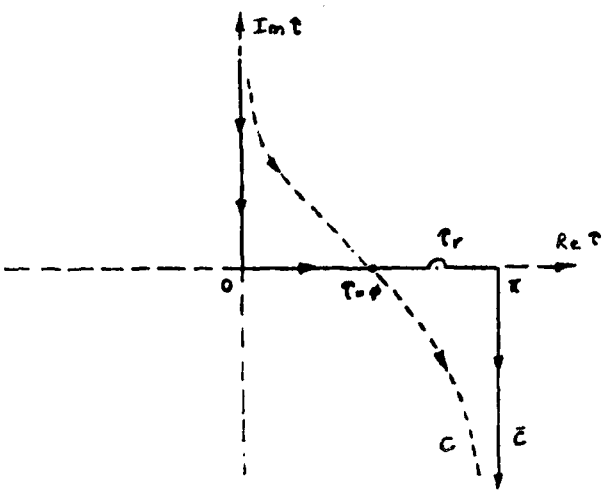


Figure-5. Deforming of path C into steepest descent path $\bar{C}$.

6. FIELD ANALYSIS AND NUMERICAL RESULTS

The total field is

$$E = E^i + E^r + E^d$$

where the reflected waves are given by (19) and the reflection coefficients R_x^e, R_z^e, R_x^m, R_z^m can easily be determined by means of (39-40) and Maxwell's equations. They are as follows:

$$R_z^e = \frac{d_3 c_1 - c_3 d_1}{d_2 c_1 - c_2 d_1},$$

$$R_x^e = \frac{c_3}{c_1} - \frac{c_2}{c_1} R_z^e,$$

$$R_x^m = -1 + \frac{k}{Z} \frac{1 + R_z^e}{k_y + k_z},$$

$$R_z^m = -1 + \frac{k}{Z_z^-} \frac{1 + R_x^e}{k_y - k_x}.$$

where

$$c_1 = 1 + \frac{Z_z^-}{k}k_y + \frac{Z_z^- k_x^2}{kk_y}, \quad c_2 = \frac{Z_z^-}{k}\frac{k_x k_z}{k_y}$$

$$c_3 = -1 - \frac{Z_z^-}{k}(k_x - k_y), \quad d_1 = \frac{Z_z^-}{k}\frac{k_x k_z}{k_y}$$

$$d_2 = 1 + \frac{Z}{k}k_y + \frac{Z k_z^2}{kk_y}, \quad d_3 = -1 + \frac{Z}{k}(k_z + k_y)$$

6.1. Diffracted Field and Diffraction Coefficient

Now, using the expressions of $M(\alpha)$ and $N(\alpha)$, the diffracted fields given by (25) and (26) can be obtained in terms of the integral expression as follows

$$\begin{bmatrix} E_x^d(y,z) \\ E_z^d(y,z) \end{bmatrix} = \int_L \frac{\kappa^2}{\Delta(\alpha)} \begin{bmatrix} Z_o Z\kappa^2 - kZ_o\Gamma & \alpha k_x Z_o Z_z^- \\ \alpha k_x & \kappa^2 - kZ_z^-\Gamma \end{bmatrix} \begin{bmatrix} L_1(\alpha) \\ L_2(\alpha) \end{bmatrix} e^{i\alpha z - i\Gamma(\alpha)y} d\alpha. \tag{119}$$

Since it is difficult to express these integrals in explicit form, it is not possible to consider the behaviour of the diffracted fields. However, in general, one is interested in the solution for $k \to \infty$ which can be obtained by applying asymptotic techniques. Here, the steepest descents method will be used to obtain the difracted field. For this purpose, it is necessary to introduce the following substitutions.

$$z = r\cos\phi, \qquad y = r\sin\phi$$

Now, changing variable

$$\alpha = -\kappa\cos\tau$$

shows the tranformation of the Fig.-3 to the Fig.-5 for the band $0 < \mathrm{Re}(\tau) < \pi$. According to the transformation, the curve C in τ-plane takes place instead of the integration line L in α-plane.

$$\begin{bmatrix} E_x^d(r,\phi,z) \\ E_z^d(r,\phi,z) \end{bmatrix} = \int_C \frac{\kappa\sin\tau}{\Delta(-\kappa\cos\tau)} \begin{bmatrix} Z_o Z\kappa^2 - kZ_o\kappa\sin\tau & -\kappa\cos\tau k_x Z_o Z_z^- \\ -\kappa\cos\tau k_x & \kappa^2 - kZ_z^-\kappa\sin\tau \end{bmatrix} \begin{bmatrix} L_1(-\kappa\cos\tau) \\ L_2(-\kappa\cos\tau) \end{bmatrix} e^{i\kappa r\cos(\phi-\tau)} d\tau. \tag{120}$$

The function inside the integrand is analytic everywhere, except that it may have some simple poles in addition to complex poles whose residues can be interpreted as surface waves departing from the junction. Now, we may find the saddle point for the integrals. For this purpose, the exponential term may be written as

$$e^{i\kappa r\cos(\phi-\tau)} = e^{i\kappa q(\tau)}$$

where

$$q(\tau) = r\cos(\phi-\tau)$$

and its first derivative gives the saddle point as $\tau = \phi$.

In steepest descents method, above type of integral path is deformed to steepest descent path $\bar{C}$ as shown in Fig.-5. When the integration evaluated along the steepest descent path, it will give the diffracted field as follows:

$$\begin{bmatrix} E_x^d(r,\phi,x) \\ E_z^d(r,\phi,x) \end{bmatrix} \approx \begin{bmatrix} D_x^d(\phi_o,\phi) \\ D_z^d(\phi_o,\phi) \end{bmatrix} \frac{e^{i\kappa r}}{\sqrt{\kappa r}}. \tag{121}$$

where the diffraction coefficients are

$$\begin{bmatrix} D_x^d(\phi_o,\phi) \\ D_z^d(\phi_o,\phi) \end{bmatrix} = \frac{\kappa\sin\phi}{\Delta(-\kappa\cos\phi)(\kappa\cos\phi+k_z)} \begin{bmatrix} Z_oZ\kappa^2 - kZ_o\kappa\sin\phi & -\kappa\cos\phi k_x Z_o Z_z^- \\ -\kappa\cos\phi k_x & \kappa^2 - kZ_z^-\kappa\sin\phi \end{bmatrix}$$
$$\begin{bmatrix} G_{U_{11}}^{-1}(-k_z) & G_{U_{12}}^{-1}(-k_z) \\ G_{U_{21}}^{-1}(-k_z) & G_{U_{22}}^{-1}(-k_z) \end{bmatrix} \begin{bmatrix} G_{L11} & G_{L12} \\ G_{L21} & G_{L22} \end{bmatrix} \begin{bmatrix} V_{o1} \\ V_{o2} \end{bmatrix} \frac{e^{i\pi/4}}{\sqrt{2\pi}}. \tag{122}$$

Fig.-6 illustrates the diffraction coefficient versus observation angle. Geometrical parameters w_1,w_2 and h are chosen properly so as to satisfy the conditions $kh << 1$ and $kw << 1$.

6.3. Surface Waves

$$\begin{bmatrix} E_x^d(y,z) \\ E_z^d(y,z) \end{bmatrix} = \int_L \frac{\kappa^2}{\Delta(\alpha)} \begin{bmatrix} Z_oZ\kappa^2 - kZ_o\Gamma & \alpha k_x Z_o Z_z^- \\ \alpha k_x & \kappa^2 - kZ_z^-\Gamma \end{bmatrix}$$
$$\begin{bmatrix} L_1(\alpha) \\ L_2(\alpha) \end{bmatrix} e^{i\alpha z - i\Gamma(\alpha)y} d\alpha. \tag{123}$$

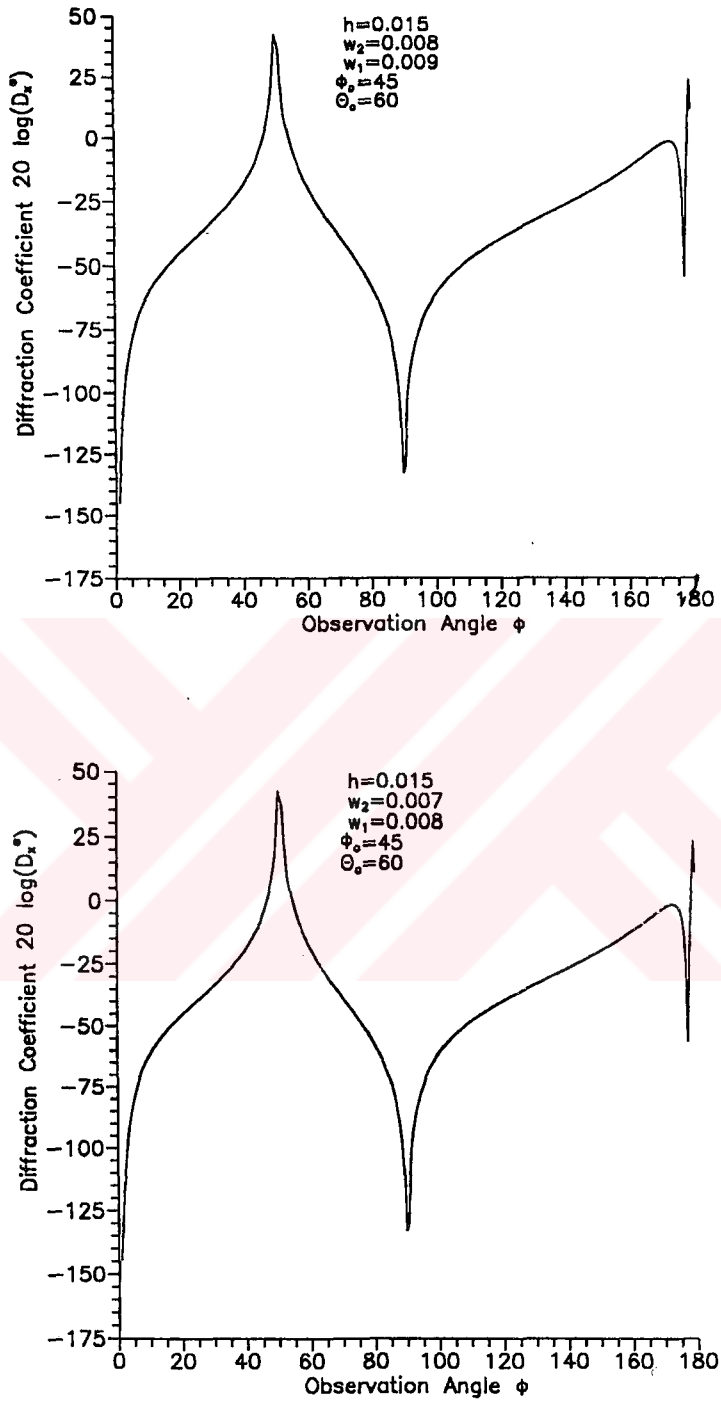


Figure-6a. Diffraction coefficient for different angles and slot width.

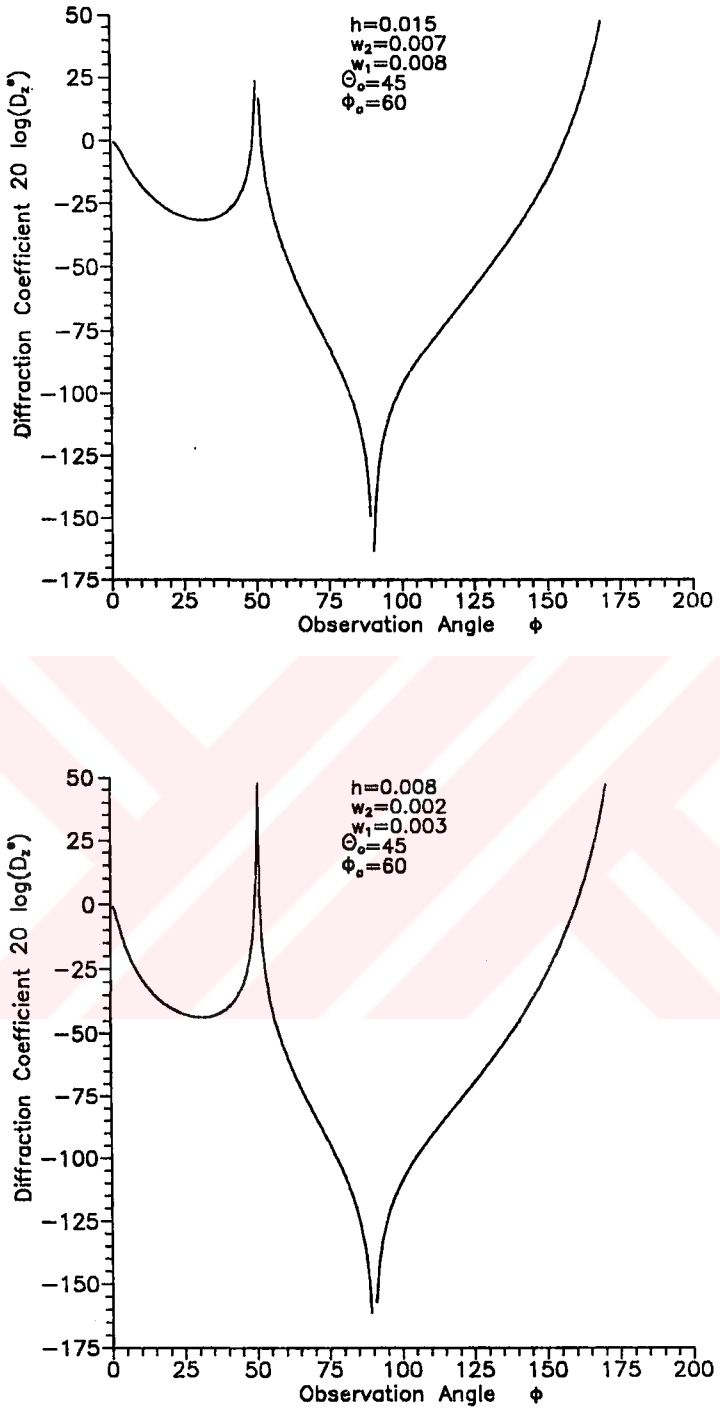


Figure-6b. Diffraction coefficient for different angles and slot width.

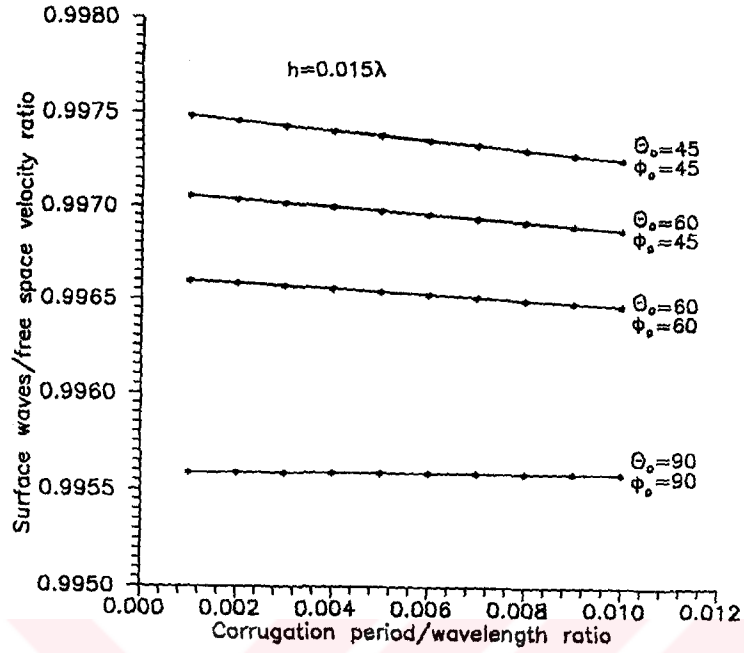


Figure-7a. Normalized Surface Wave Velocity.

The above integral expression have four complex poles, come from terms $\lambda_{1L}(\alpha)$ and $\lambda_{2L}(\alpha)$, which can be interpreted as surface waves departing from the junction since these zeros are the poles of the field expressions. Two of these surface wave poles are always far from the saddle point and could be associated with a possible higher-order mode at the junction. Therefore they can be neglected during the analysis. The surface waves coming from the other two poles can easily be determined by the residue contribution of the expression given by (25). It is expected that these surface waves are proceeding in O_{xz} plane with the angle with respect to z-axis ϕ_s and attenuating in y direction exponentially. The main effect on the surface waves, as expected, are the geometrical parameters of the structure which are given as corrugation depth h, corrugation widths w_1,w_2. The velocity and power of these surface waves varies with respect to the incidence angle also.

In general, a surface wave which is excited at the junction, propagating in the O_{xz} plane can be expressed as

$$f_s = A_s \exp(-ik_x x + i\alpha_s z - i\Gamma(\alpha_s)y). \tag{124}$$

where A_s is the amplitude of the surface wave determined from residue contribution

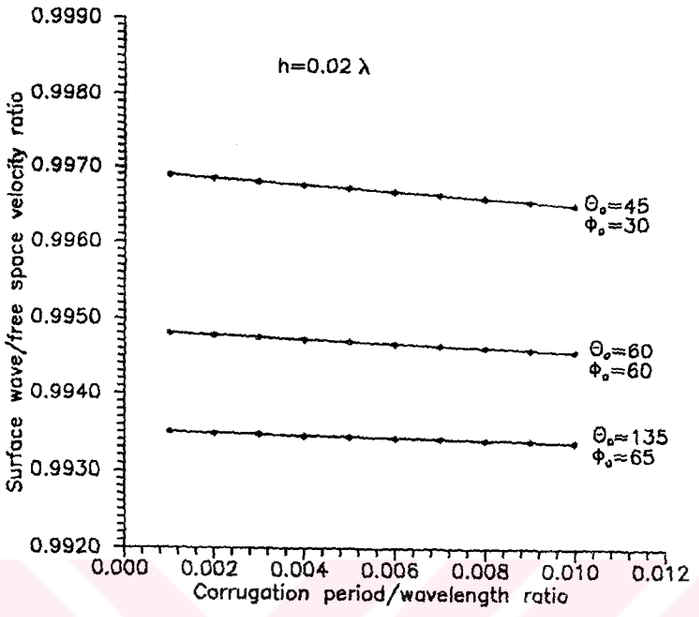


Figure-7b. Normalized Surface Wave Velocity.

of the field expression, k_x,α_s are real values and define the propagation constant in the direction of x, z, respectively. $\mathrm{Im}\{\Gamma(\alpha_s)\}$ gives the attenuation in y direction.

The ratio of surface wave velocity to that of free space can be given as follows:

$$\frac{v_s}{c} = \frac{k}{\sqrt{{\alpha_s}^2 + {k_x}^2}} \tag{125}$$

and the propagation direction of this wave will be

$$\phi_s = \arctan(k_x/\alpha_s). \tag{126}$$

The velocity of the surface waves relative to that of free space versus incidence angle and slot width is shown in Fig.-7.

7. CONCLUSIONS

Scattering mechanism related with the junction of two corrugated half planes has been investigated and some results concerned with the diffraction coefficient and the surface wave velocity has been given.

Fig.-6 illustrates the variation of the diffraction coefficient given by (122) with respect to the observation angle for different geometrical parameters of the structure and incident angles. The amplitude of the incident field is taken as unity. As expected, since the diffracted field given by (121) is not uniform, the diffraction coefficient will take very large values in the transition regions.

The velocity of surface waves excited at the junction is shown in Fig-.7. The surface wave velocity increases when either the corrugation width or corrugation height decrease or both decresase. There is a good aggreement between these results and those given in reference [2]. As a result, the stucture under consideration is suitable to use a slow-wave structure in microwave applications.

SUMMARY

Corrugated surfaces are widely used in microwave applications. Discontinuity formed by two corrugated surfaces with different corrugation widths may be very interesting for waveguides. Weinstein [8] has shown that higher order impedance boundary conditions can be employed to simulate corrugated surfaces.

In this study, scattering mechanism of plane electromagnetic wave at the junction of two corrugated half-planes is investigated. First, standard impedance boundary conditions are used to simulate anisotropic corugated half-planes. So the problem is reduced to the discontinuity formed by two anisotropic impedance half planes. The problem is formulated by Fourier Transform technique and all tangential field components are derived from Hertz vector fields. Using Leontovich impedance boundary conditions, four functional equations are obtained and this system of equation is written in Matrix Wiener-Hopf system. The system is also solved in a manner given by [9] and this is carried in Appendix-I.

The solution of a Matrix Wiener-Hopf system is identical to the solution procedure of a scalar Wiener-Hopf equation. The classical Wiener-Hopf procedure requires the factorization of a kernel matrix which can be factorized into regular two matrices in lower and upper complex half planes. When the kernel matrix is a scalar, the factorization can easily be carried out by means of Cauchy's theorem. Since a matrix do not form a commutative algebra, standard procedure cannot be operated generally. This is the main source of difficulty in solving matrix Wiener-Hopf equations. Another difficulty comes from the exponential growth of the kernel matrix and this is achieved by introducing a polynomial matrix to ensure the regularity condition. Daniele-Khrapkov method is applicable for this kind of problem and here, this method is used.

After the necessary spectral coefficients are determined, the resultant integral form of diffracted field is evaluated by the method of steepest descent path (SDP) for large k. This method is very powerful for the integrals to be evaluated in high-frequency electromagnetic techniques. Diffraction coefficients and diffracted fields are determined and the results are given.

The residue contribution of the expression inside the resultant integral gives the surface waves. The effect of the geometrical parameters of the structure on the velocity of the surface waves is very important and this is considered. It is shown

that there is good agreement between the results in literature and those which we obtained. This kind of structure can be used as slow-wave structure in microwave applications.

ÖZET

Oluklu yapılar mikrodalga uygulamalarında geniş bir şekilde kullanılmaktadır. Farklı oluk periyotlu iki oluklu yarı düzlem eklemi dalga kılavuzlarında sık karşılaşılacak yapılardandır. Bu süreksizlik noktasında elektromanyetik dalgaların davranışı ilginç olacaktır. İlk olarak Weinstein [8], oluklu yapıların yüksek mertebeden sınır koşulları ile simüle edilebileceğini göstermiştir.

Bu çalışmada, farklı oluk genişlikli iki oluklu yarı düzlem ekleminden düzlem elektromanyetik dalgaların saçılması incelenmiştir. Bu amaçla, Weisntein tarafından verilen sınır koşullarına dayanarak standard empedans sınır koşulları ile oluklu yapılar simüle edilmektedir. Böylece problem anizotrop iki yarı düzlem ekleminden saçılma problemine indirgenmiştir. Daha sonra Fourier Transform Tekniği ile formülasyon yapılmış ve tüm teğetsel alan bileşenleri manyetik ve elektrik Hertz vektörleri cinsinden elde edilmiştir. Leontovich sınır koşulları ile dört adet fonksiyonel denklem elde edilmiş, bu denklem takımı Matris Wiener-Hopf metodu ile çözülebilecek forma indirgenmiştir.

Matris Wiener-Hopf sistemi skaler Wiener-Hopf sistemi ile aynı çözüm prensibine dayanır. Klasik Wiener-Hopf denklemi, davranışı hakkında bir şey bilinmeyen bir çekirdek matrisin, alt ve üst yarı düzlemlerde regüler olan iki ayrı matrise faktörize edilmesi prensibine dayanır. Çekirdek matris skaler olduğu zaman, Cauchy teoremi yardımıyla faktörizasyon işlemi kolayca yapılabilir. Fakat genelde matrisler komütatif özellik göstermediklerinden, Matris Wiener-Hopf sistemi kolayca çözülemez. Bu çalışmada, Daniele-Khrapkov metodu ile Matris Wiener-Hopf sistemi çözülmüştür. Çözüme giderken karşılaşılan diğer bir zorlukta çekirdek matrisin fiziksel olarak bir anlamı olmayan aşırı yüksek değerler almasıdır. Bu zorlukta, bir polinom matris yardımıyla giderilmektedir.

Alan ifadelerinde bulunan spektral fonksiyonlar böylece hesaplanarak integral ifadesi elde edilmiştir. Elde edilen integral çok büyük k değerleri için en dik iniş çizgisi yöntemi ile değerlendirilmiştir. Elde edilen kırınan alan ifadesinin geliş açısına göre değişimi incelenmiş ve GTD (Geometrical Theory of Diffraction) uygulaması olarak kırınım katsayıları hesaplanmıştır. Buna ilaveten, eklemde uyarılabilecek yüzey dalgaları da incelenmiştir. Sözkonusu yüzey dalgaları elde edilen integral ifadesinin rezidü katkısı ile elde edilmektedir. Oluklu yapının geometrik parametrelerinin yüzey dalga hızına etkisi de incelenmiştir.

APPENDIX I

Since one of the elements of $\mathbf{G}(\alpha)$ is zero, it should be noted that matrix Wiener-Hopf system given by (43) can be solved by successive elimination yielding to a scalar Wiener-Hopf equation. Now let us write (43) in following form

$$\frac{1}{\Delta(\alpha)}\begin{bmatrix} W(\alpha) & \alpha k_x(Z_z^- Z_z^+) \\ 0 & T(\alpha) \end{bmatrix}\begin{bmatrix} L_1(\alpha) \\ L_2(\alpha) \end{bmatrix} = \begin{bmatrix} U_1(\alpha) \\ U_2(\alpha) \end{bmatrix} + \begin{bmatrix} V_1(\alpha) \\ V_2(\alpha) \end{bmatrix}$$

where

$$T(\alpha) = \Delta(\alpha) = Z\kappa^2 + ka\Gamma(\alpha) + Z_z^-(k^2 - \alpha^2)$$
$$W(\alpha) = Z\kappa^2 + kb\Gamma(\alpha) + Z_z^+(k^2 - \alpha^2).$$

It may be written that $L_2(\alpha) = V_2(\alpha)$ and $U_2(\alpha)$. The functional equation is

$$\frac{W(\alpha)}{T(\alpha)}L_1(\alpha) + \frac{\alpha k_x(Z_z^- - Z_z^+)}{T(\alpha)}L_2(\alpha) = U_1(\alpha) + V_1(\alpha)$$

rearranging the equation gives the the Wiener-Hopf system as

$$\frac{T_U(\alpha)}{W_U(\alpha)}U_1(\alpha) - \frac{W_L(\alpha)}{T_L(\alpha)}L_1(\alpha) = f(\alpha)$$

where

$$f(\alpha) = \frac{T_U(\alpha)}{W_U(\alpha)}\left\{V_1(\alpha) - \frac{\alpha k_x(Z_z^- - Z_z^+)V_2(\alpha)}{T(\alpha)}\right\}.$$

Factorization of $f(\alpha)$ as

$$f(\alpha) = f_U(\alpha) + f_L(\alpha)$$

where

$$f_U(\alpha) = \frac{1}{2\pi i}\int_{\mathcal{L}_U} f(t)\frac{dt}{(t-\alpha)},$$
$$f_L(\alpha) = -\frac{1}{2\pi i}\int_{\mathcal{L}_L} f(t)\frac{dt}{(t-\alpha)}.$$

Considering the comlex α-plane and the integration lines defined in the plane as shown in Fig.-3.

$$f_L(\alpha) = \frac{T_U(-k_z)}{W_U(-k_z)}\left\{V_1(\alpha) + \frac{k_x k_z(Z_z^- - Z_z^+)V_2(\alpha)}{T(-k_z)}\right\}.$$

is found.

So, the solution is obtained as

$$L_1(\alpha) = -\frac{T_L(\alpha)}{W_L(\alpha)} \frac{T_U(-k_z)}{W_U(-k_z)} \left\{ V_1(\alpha) + \frac{k_x k_z (Z_z^- - Z_z^+) V_2(\alpha)}{T(-k_z)} \right\}.$$

The unkown spectral functions $M(\alpha)$ and $N(\alpha)$ can easily be determined from (118).

APPENDIX II

This computer program computes the difraction coefficient for E_x^d, E_z^d.

```
COMPLEX GLM11,GLM12,GLM22,GUM12,GUM11,GUM22,VO1,VO2
COMPLEX Z,ZZM,DEL1,DEL2,DEL,C1,C2,C21,L2,L1,M
COMPLEX DIF2,CO2
REAL*8 DIF1,GAM,CS1,QM,CO1,DIF
REAL*8 PHI,THEO,FIR,PHIO,WP,H,PI,WM,K,KX,KAP,ALP
CHARACTER*8 DOS
WRITE(*,*)'ENTER OUTPUT FILE NAME='
READ(*,18) DOS
OPEN(6,FILE=DOS,FORM='FORMATTED',STATUS='NEW')
18 FORMAT(A8)
WRITE(*,*)'ENTER THO,FIO,WP,WM,H ='
READ(*,*)THEO,PHIO,WP,WM,H
PI=4.0*ATAN(1.0)
THEO=THEO*PI/180
PHIO=PHIO*PI/180
PHI=PI/180
K=2.*PI
KX=K*COS(THEO)*SIN(PHIO)
KZ=K*COS(PHIO)
KAP=SQRT(K**2-KX**2)
CS1=COSH(PI*H/WM)
QM=(WM/PI)*DLOG(CS1)
Z=CMPLX(0.,-1.)*K*H
ZZM=CMPLX(0.,-1.)*K*(H-QM)
A=-(Z*ZZM+1)
CALL GUM(THEO,PHIO,-KZ,WP,WM,H,GUM11,GUM12,GUM22)
DO 14 I=1,628
ALP=-KAP*COS(PHI)
GAM=SQRT(KAP**2-ALP**2)
CO1=-K*377*GAM+Z*KAP**2
CO2=ALP*KX*ZZM
DEL1=Z*KAP**2+A*K*GAM
DEL2=ZZM*(K**2-ALP**2)
DEL=KAP**2*(DEL1+DEL2)
CALL VO(PHI,THEO,PHIO,WP,WM,H,VO1,VO2)
```

```
CALL GLM(THEO,PHIO,ALP,WP,WM,H,GLM11,GLM12,GLM22)
C1=GUM11*GLM11*VO1
C21=GUM11*GLM12+GUM12*GLM22
C2=C21*VO2
L1=C1+C2
L2=GUM22*GLM22*VO2
M=(L1*CO1+CO2*L2)/DEL
DIF1=KAP*SIN(PHI)*M
DIF2=DIF1/SQRT(2*PI)
DIF=20*ALOG(CABS(DIF2))
FIR=PHI*180/PI
WRITE(6,*)FIR,DIF
PHI=PHI+0.01
14 CONTINUE
END
************************************************************************
SUBROUTINE VO(FI,THO,FIO,WP,WM,H,VO1,VO2)
COMPLEX ZN,ZZPN,ZZMN,HT,T,C11,C12,C1,C2,C3,T1,D1
COMPLEX D2,D3,RZE1,RZE2,RZE,RXE
COMPLEX RZM,P1,P2,VO1,VO2,RXM,R1,R2
REAL*8 PI,K,KX,KY,KZ,QP,H,WP,WM,QM,ALP,CS
REAL*8 FI,FIO,THO,KAP,CS1,RO,RRO,DEN
PI=4.*ATAN(1.)
K=2.*PI
KX=K*SIN(FIO)*COS(THO)
KY=K*SIN(FIO)*SIN(THO)
KZ=K*COS(FIO)
KAP=SQRT(K**2-KX**2)
ALP=-KAP*COS(FI)
CS=COSH(PI*H/WP)
QP=(WP/PI)*DLOG(CS)
ZN=CMPLX(0.,-1.)*K*H
ZZPN=CMPLX(0.,-1.)*K*(H-QP)
CS1=COSH(PI*H/WM)
QM=(WM/PI)*DLOG(CS1)
ZZMN=CMPLX(0.,-1.)*K*(H-QM)
HT=ZZPN/K
T=ZZMN/K
C11=T*KY
C12=T*KX**2/KY
```

```
C1=1+C11+C12
C2=T*KX*KZ/KY
C3=-1-T*(KX-KY)
T1=ZN/K
D1=T1*KX*KZ/KY
D2=1+T1*KY+T1*KZ**2/KY
D3=-1+T1*(KY+KZ)
RZE1=D3*C1-C3*D1
RZE2=D2*C1-C2*D1
RZE=RZE1/RZE2
RXE=(C3-C2*RZE)/C1
RO=(KZ+KY)
RRO=(-KX+KY)
R1=1+RXE
R2=1+RZE
RZM=-1+R1/(T*RRO)
RXM=-1+R2/(T1*RO)
DEN=ALP+KZ
P1=(1+RZM)*(KY-KX)
P2=(1+RXM)*(KY+KZ)
VO1=(1+RXE-HT*P1)/DEN
VO2=(1+RZE-T1*P2)/DEN
END
*******************************************************
SUBROUTINE GLM(THO,FIO,ALP,WP,WM,H,GLM11,GLM12,GLM22)
COMPLEX Z,ZZM,ZZP,A,B,L1,LO,L,M,PL11,PL22,PU11,PU22
COMPLEX PL12,PU12,S,C1,C2,C3,P11,P12,P21,P22,GL11,GL22
COMPLEX P31,P32,P33,GL12,GLM11,GLM22,GLM12
REAL*8 PI,K,KX,FIO,THO,KAP,ALP,WP,WM,H,CS
REAL*8 CS1,QP,QM
PI=4.*ATAN(1.)
K=2.*PI
KX=K*SIN(FIO)*COS(THO)
KAP=SQRT(K**2-KX**2)
CS=COSH(PI*H/WP)
CS1=COSH(PI*H/WM)
QP=(WP/PI)*DLOG(CS)
QM=(WM/PI)*DLOG(CS1)
Z=CMPLX(0.,-1.)*K*H
ZZP=CMPLX(0.,-1.)*K*(H-QP)
```

```
ZZM=CMPLX(0.,-1.)*K*(H-QM)
A=-(Z*ZZM+1)
B=-(Z*ZZP+1)
L1=A*ZZP-B*ZZM
LO=0.5*(Z*KAP**2*(A-B)+L1*K**2)
L=LO-ALP**2*L1*0.5
M=A*ALP*KX*(ZZM-ZZP)
CALL LAMP(THO,FIO,ALP,WP,WM,H,PL11,PL22,PU11,PU22,
S,PL12,PU12)
CALL C123(THO,FIO,ALP,WP,WM,H,C1,C2,C3)
P11=A*C2
P12=C1*C3*A*L
GL11=C1*PL11*(P11+P12)
P21=B*C2
P22=B*C3*L
GL22=C1*PL22*(P21-P22)
P31=C2*PL12
P32=C3*L*PL12
P33=C3*M*PL22
GL12=C1*A*(P31+P32-P33)
GLM11=1/GL11
GLM22=1/GL22
GLM12=-GL12/(GL11*GL22)
END
************************************************************
SUBROUTINE GUM(THO,FIO,ALP,WP,WM,H,GUM11,GUM12,GUM22)
COMPLEX Z,ZZM,ZZP,A,B,L,M,PL11,PL22,PU11,PU22
COMPLEX PL12,PU12,C1,C2,C3,P11,P12,P21,P22,GL11,GL22
COMPLEX P31,P32,P33,GL12,GUM11,GUM22,GUM12
REAL*8 PI,K,KX,FIO,THO,ALP,WP,WM,H,CS
REAL*8 CS1,QP,QM
PI=4.*ATAN(1.)
K=2.*PI
KX=K*SIN(FIO)*COS(THO)
CS=COSH(PI*H/WP)
CS1=COSH(PI*H/WM)
QP=(WP/PI)*DLOG(CS)
QM=(WM/PI)*DLOG(CS1)
Z=CMPLX(0.,-1.)*K*H
ZZP=CMPLX(0.,-1.)*K*(H-QP)
```

```
ZZM=CMPLX(0.,-1.)*K*(H-QM)
A=-(Z*ZZM+1)
B=-(Z*ZZP+1)
M=-A*ALP*KX*(ZZM-ZZP)
CALL LAMP(THO,FIO,-ALP,WP,WM,H,PL11,PL22,PU11,PU22,
L,PL12,PU12)
CALL C123(THO,FIO,-ALP,WP,WM,H,C1,C2,C3)
P11=A*C2
P12=C1*C3*A*L
GL11=C1*PL11*(P11+P12)
P21=B*C2
P22=B*C3*L
GL22=C1*PL22*(P21-P22)
P31=C2*PL12
P32=C3*L*PL12
P33=C3*M*PL22
GL12=C1*A*(P31+P32-P33)
GUM11=1/GL11
GUM22=1/GL22
GUM12=-GL12/(GL11*GL22)
END
*************************************************************
SUBROUTINE C123(THO,FIO,ALP,WP,WM,H,C1,C2,C3)
COMPLEX LAMP1L,LAMP2L,LAMP1U,LAMP2U
COMPLEX LAM1L,LAM2L,LAM1U,LAM2U,L,DEL,DELL,DELU
COMPLEX C1,C21,C2,C3,CO,D2,PL12,PU12
REAL*8 PI,D1,ALP,WP,WM,H
REAL*8 K,KX,KAP,FIO,THO
PI=4.*ATAN(1.)
K=2.*PI
KX=K*SIN(FIO)*COS(THO)
KAP=SQRT(K**2-KX**2)
D1=SQRT(K*(KAP-ALP))
CALL LAMP(THO,FIO,ALP,WP,WM,H,LAMP1L,LAMP2L,LAMP1U,
LAMP2U,L,PL12,PU12)
CALL LAM(THO,FIO,WP,ALP,H,LAM2U,LAM2L,DELU,DEL)
CALL LAM(THO,FIO,WM,ALP,H,LAM1U,LAM1L,DELU,DELL)
D2=LAM1L*LAM2L/(LAMP1L*LAMP2L)
CO=D1*CSQRT(D2)
C1=CO/DELL
```

```
C21=LAM1L*LAMP2L/(LAM2L*LAMP1L)
C2=0.5*(CSQRT(C21)+1/CSQRT(C21))
C3=(0.5/L)*(CSQRT(C21)-1/CSQRT(C21))
END
************************************************************
SUBROUTINE LAMP(THO,FIO,ALP,WP,WM,H,LAMP1L,LAMP2L,LAMP1U,LAMP2U.
COMPLEX E,F,Z,ZZP,A,B,NU,ZZM,L1,D12,D11,D1
COMPLEX DEL,L,DEL1,DEL2,LAMP2U,LAMP2L,TO
COMPLEX LO,K1,K2,LAMP1L,LAMP1U,SIG1,SIG2,D21,D22,D2
REAL*8 PI,QP,H,WP,CS,CS1,WM,QM,RENU
REAL*8 K,KX,KAP,FIO,THO,ALP,T1
PI=4.*ATAN(1.)
K=2.*PI
KX=K*SIN(FIO)*COS(THO)
KAP=SQRT(K**2-KX**2)
CS=COSH(PI*H/WP)
CS1=COSH(PI*H/WM)
QP=(WP/PI)*DLOG(CS)
QM=(WM/PI)*DLOG(CS1)
Z=CMPLX(0.,-1.)*K*H
ZZP=CMPLX(0.,-1.)*K*(H-QP)
ZZM=CMPLX(0.,-1.)*K*(H-QM)
A=-(Z*ZZM+1)
B=-(Z*ZZP+1)
E=A*ZZP-B*ZZM
F=Z*(B-A)*KAP**2
NU=CSQRT(F/E-K**2)
RENU=AIMAG(NU)
T1=0.5*ATAN(K/RENU)
TO=NU*TAN(T1)
DEL1=(1-E*0.5)*TO**2
DEL2=0.5*(E*K**2-F)
DEL=DEL1+DEL2
LO=0.5*(E*K**2-F)
L1=-E*0.5
L=(LO+ALP**2*L1)
K1=1-L1
K2=1+L1
SIG1=CSQRT((DEL+LO)/K1)
SIG2=CSQRT((DEL-LO)/K2)
```

```
IF (AIMAG(SIG1).GT.0.) THEN
LAMP1L=K1*(SIG1-ALP)
LAMP1U=K1*(SIG1+ALP)
ELSE
LAMP1L=K1*(SIG1+ALP)
LAMP1U=K1*(SIG1-ALP)
ENDIF
IF (AIMAG(SIG2).GT.0.) THEN
LAMP2L=K2*(SIG2-ALP)
LAMP2U=K2*(SIG2+ALP)
ELSE
LAMP2L=K2*(SIG2+ALP)
LAMP2U=K2*(SIG2-ALP)
ENDIF
D21=B*KX*(ZZM-ZZP)
D22=(SIG1+SIG2)*SQRT(K2)
D2=SIG2*D21/D22
D11=B*KX*(ZZP-ZZM)
D12=(SIG1+SIG2)*SQRT(K1)
D1=SIG1*D11/D12
END
*****************************************************************
SUBROUTINE LAM(THO,FIO,ALP,W,H,LAMU,LAML,DU,DL)
COMPLEX Z,ZZ,GAM1,GAM2,SIG1,SIG2,ETA1,ETA2,KAP1L
COMPLEX KAP1U,KAP2L,KAP2U,C1,LAMU,LAML,KAPPA,AW
COMPLEX DENU,DENL,C11,CO,DL,DU
REAL*8 PI,Q,H,W,ALP,CS
REAL*8 FIO,THO,K,KAP,KX
PI=4.*ATAN(1.)
K=2.*PI
KX=K*SIN(FIO)*COS(THO)
KAP=SQRT(K**2-KX**2)
CS=COSH(PI*H/W)
Q=(W/PI)*DLOG(CS)
Z=CMPLX(0.,-1.)*K*H
ZZ=CMPLX(0.,-1.)*K*(H-Q)
AW=-(Z*ZZ+1)
GAM1=AW*K/ZZ
GAM2=K**2-KAP**2-(Z/ZZ)*KAP**2
SIG1=0.5*(GAM1+CSQRT(GAM1**2+4*GAM2))
```

```
SIG2=0.5*(GAM1-CSQRT(GAM1**2+4*GAM2))
ETA1=KAP/SIG1
ETA2=KAP/SIG2
KAP1L=KAPPA(ETA1,KAP,-ALP)
KAP1U=KAPPA(ETA1,KAP,ALP)
C11=AW*ETA1*ETA2*K
C1=CSQRT(ZZ/C11)
KAP2L=KAPPA(ETA2,KAP,-ALP)
KAP2U=KAPPA(ETA2,KAP,ALP)
DENU=KAP1U*KAP2U
DENL=KAP1L*KAP2L
LAMU=C1*SQRT(KAP+ALP)/DENU
LAML=C1*SQRT(KAP-ALP)/DENL
CO=CSQRT(ZZ/(ETA1*ETA2))
DL=CO*(KAP-ALP)/DENL
DU=CO*(KAP+ALP)/DENU
END
****************** function of kappa ******************
FUNCTION KAPPA(KSI,XKAP,ALP)
C complex split function X+(ksi,cos(ang))
COMPLEX KAPPA,KSI,S,ACO,ACOF,AR1,AR2,F
REAL*8 PI,ANG,XKAP,ALP
PI=3.141592654D0
ANG=ACOS(-ALP/XKAP)
ACO=ACOF(1./KSI)
S=CSQRT(2./KSI)*SIN(ANG/2.)
AR1=PI-ANG+ACO
AR2=PI-ANG-ACO
KAPPA=S*F(AR1)*F(AR2)
RETURN
END
C Function f(ar)
FUNCTION F(AR)
COMPLEX AR,F,B,C,D,XMAL
REAL*8 MALP,A
MALP=0.965628448D0
A=2.**(3./4.)
B=1.+SQRT(2.)*CCOS(AR/2.)
C=XMAL(AR)/MALP
D=C*C
```

```
F=A*D/B
RETURN
END

****************** function of maliuz ********************
function xmal(ze)
complex xmal,ze,z1,pr,ccs,op
real*8 pi2,xi,yi,xo,yo,xb,t,pf
pi2=2*atan(1.0)
xi=real(ze)
yi=aimag(ze)
mconj=0
if ((xi.le.0.).and.(yi.le.0.)) then
xo=-xi
yo=-yi
elseif ((xi.le.0.).and.(yi.ge.0.)) then
xo=-xi
yo=yi
mconj=1
elseif ((xi.ge.0.).and.(yi.le.0.)) then
xo=xi
yo=-yi
mconj=1
else
xo=xi
yo=yi
endif
m=0
xb=xo
if (xo.gt.pi2) then
if (xo.gt.(7*pi2)) then
xb=xo-8*pi2
elseif (xo.gt.(5*pi2)) then
xb=xo-6*pi2
elseif (xo.gt.(3*pi2)) then
xb=xo-4*pi2
else
xb=xo-2*pi2
endif
m=1
```

```
endif
t=alog(2.)
pf=0.965628448D0**2
z1=xb+cmplx(0.,1.)*yo
ccs=ccos(z1*0.25-cmplx(0.,1.)*t*0.25)
if ((yo.ge.0.).and.(yo.le.4.6)) then
xmal=1.0d0-0.01390038813d0*(z1**2)
elseif ((yo.gt.4.6).and.(yo.le.8.)) then
pr=((cmplx(0.,1.)*z1)/(4*pi2))*cexp(cmplx(0.,1.)*z1)
xmal=1.053025289d0*csqrt(ccs)*cexp(pr)
else
xmal=1.053025289d0*csqrt(ccs)
endif
10 if (m.eq.1) then
z1=z1+2.*pi2
op=ccos((z1-cmplx(pi2,0.))*0.25)
xmal=pf*op/xmal
endif
if (real(z1).lt.xo) go to 10
if (mconj.eq.1) xmal=conjg(xmal)
end
C
c computing the inverse cosine of complex number
function acof(eta)
complex eta,cj,acof
real re,ae,rep,rem,aa,bb,sgn,raa,pi
pi=3.14159265358979323846
cj=(0.,1.0d0)
re=real(eta)
ae=aimag(eta)
rep=re+1
rem=re-1
aa=.5*(sqrt(rep*rep+ae*ae)+sqrt(rem*rem+ae*ae))
bb=.5*(sqrt(rep*rep+ae*ae)-sqrt(rem*rem+ae*ae))
if(ae.ne.0.d0)then
sgn=ae/abs(ae)
else
sgn=1.d0
endif
raa=aa*aa-1.
```

```
if(raa.lt.1.e-6)raa=0.
acof=asin(bb)+cj*alog(aa+sqrt(raa))*sgn
acof=.5*pi-acof
return
end
```

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RESUME

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